## Chem 170

# Stoichiometric Calculations 

Module One

Units, Scientific Notation, Significant Figures and Dimensional Analysis

## Introduction to Module One

Look at this piece of paper and list some of its characteristics. Many of the items on your list will be qualitative because they use descriptive phrases to describe the paper's qualities. Examples of qualitative descriptions include "the paper is white," "the paper is rectangular" and "the paper has funny black marks all over it." Other characteristics are quantitative; that is, they include numbers and units in their descriptions. For example, "the paper is 11 inches long" and "there are 5 sentences in this paragraph."

Chemists frequently make use of qualitative and quantitative information. In our introductory courses in organic chemistry, inorganic chemistry, and biochemistry, for example, you will learn how chemists determine a molecule's structure (the physical connections between atoms and the resulting 3-dimensional shape) and how this qualitative property affects its chemical behavior. Chemists also use quantitative information to characterize chemical reactivity. For example, some reactions proceed very quickly, while others are very slow. A reaction's rate, therefore, provides a quantitative measure of how quickly it occurs.

## Objectives for Module One

This module covers the basic tools for working with quantitative information, including the standard units used in chemistry, scientific notation, the importance of significant figures when expressing the result of a calculation and the use of dimensional analysis when converting between units. In completing this module you will master the following specific objectives:

- to recognize the units of the SI system
- to be able to express SI units with different prefixes
- to covert between the Fahrenheit, Celsius and Kelvin temperature scales
- to calculate the density of an object
- to express numbers using scientific notation
- to state the number of significant figures in a measurement
- to express the result of a calculation to the correct number of significant figures
- to use dimensional analysis to convert between units


## Units of Measurement

If you measure the length of this page you might report your answer as 11 inches or as 27.9 centimeters, depending upon which scale you use. Having different units for length is at worst mildly annoying. As long as we know the relationship between units (e.g. 1 inch is equivalent to 2.54 cm ) we can easily convert between them. Of greater concern to scientists is knowing a unit's accuracy. Defining one unit in terms of another unit doesn't help. We know that there are 12 inches in a foot, but what is a foot? It should be clear that a measurement is poorly defined if we cannot trace it to an absolute standard that is acceptable to all scientists.

SI Units. At one time the standard unit of length was the cubit, defined as the length of a person's forearm from the elbow to the tip of his or her middle finger. Defining a cubit in this manner makes it a relative measure of length because each individual's arm is of different length. The standard scientific measurement units used today are those established in 1960 by the General Conference of Weights and Measures, more commonly known as the Système Internationale d'Unités, or SI units. Table 1 lists the seven base SI units and their symbols. If you're interested, the National Institute of Standards and Technology web-site (http://www.physics.nist.gov/cuu/Units) provides a historical overview of these base units, including the current absolute standard for each. ${ }^{\dagger}$

Table 1. Base SI Units

| Quantity | Name of SI Unit | Symbol |
| :---: | :---: | :---: |
| length | meter | m |
| mass | kilogram | kg |
| time | second | s |
| electric current | ampere | A |
| temperature | Kelvin | K |
| amount of substance | mole | mol |
| luminous intensity | candela | cd |

All other SI units can be derived from these base units. For example, force (F), which is the product of an object's mass and its acceleration, has units of $\mathrm{kg} \mathrm{m} / \mathrm{s}^{2}$ (or $\mathrm{kg} \mathrm{m} \mathrm{s}^{-2}$ ).

The base SI unit for mass differs from the other base SI units because it contains two parts. The first part, kilo-, is a prefix meaning 1000, and the second part, gram, is a smaller unit of mass. The use of prefixes is common in the SI system; for example, a nanosecond (ns) is 0.000000001 seconds and a kilometer (km) is 1000 meters. Table 2 lists many of the most commonly used prefixes.

[^0]Table 2. Prefixes Used in the SI System

| Prefix | Symbol | Meaning | Prefix | Symbol | Meaning |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Tera- | T | $1,000,000,000,000$ | Centi- | c | 0.01 |
| Giga- | G | $1,000,000,000$ | Milli- | m | 0.001 |
| Mega- | M | $1,000,000$ | Micro- | $\mu$ | 0.000001 |
| Kilo- | k | 1,000 | Nano- | n | 0.000000001 |
| Deci- | d | 0.1 | Pico- | p | 0.000000000001 |

SI Units of Particular Importance to Chemists. Of the seven base units, the most important for us are length, mass, time, temperature, and amount of substance. Other units derived from these base units are volume, density, pressure, and concentration. Several of these units are briefly considered here; others are discussed in later modules.

Length. Atoms are very small; thus, chemists often express length in nanometers or picometers. For example, the radius of a helium atom is 50 picometers ( pm ) and the distance between two carbon atoms in ethanol is 0.15 nanometers (nm). An older unit for expressing length is the angstrom $(\AA)$, which is equivalent to 100 pm or 0.00000001 cm .

Mass. Chemists use the gram, not the kilogram, as the base SI unit.
Time. In chemistry the most common use of time is measuring the speed of a reaction. Fast reactions are reported using the second as the base unit. It is not uncommon, for example, to study reactions occurring on a millisecond or nanosecond time-scale. Slower reactions, however, usually are measured in minutes or hours instead of kiloseconds.

Temperature. Although Kelvin is the SI unit for temperature, there are two other common non-SI temperature scales. The Fahrenheit scale $\left({ }^{\circ} \mathrm{F}\right)$, which is used in daily life, defines the boiling point and freezing point of water as $212^{\circ} \mathrm{F}$ and $32^{\circ} \mathrm{F}$, respectively. The Celsius scale $\left({ }^{\circ} \mathrm{C}\right)$, which is commonly used in the laboratory, assigns $100^{\circ} \mathrm{C}$ and $0^{\circ} \mathrm{C}$ to, respectively, the boiling point and freezing point of water. The following equations allow the conversion between these two units ${ }^{\dagger}$

$$
{ }^{\circ} \mathrm{F}=\frac{9}{5}\left({ }^{\circ} \mathrm{C}\right)+32 \quad{ }^{\circ} \mathrm{C}=\frac{5}{9}\left({ }^{\circ} \mathrm{F}-32\right)
$$

Example 1. A child running a fever has a temperature of $100.6^{\circ} \mathrm{F}$. Express this temperature in ${ }^{\circ} \mathrm{C}$.

Solution. Using the equation for converting ${ }^{\circ} \mathrm{F}$ to ${ }^{\circ} \mathrm{C}$ gives

$$
{ }^{\circ} \mathrm{C}=\frac{5}{9}\left({ }^{\circ} 100.6-32\right)=38.1^{\circ} \mathrm{C}
$$

[^1]Example 2. A bank's "time/temperature" display is not working properly, showing the temperature only in ${ }^{\circ} \mathrm{C}$. What is the temperature in ${ }^{\circ} \mathrm{F}$ if the display reads $-4.5^{\circ} \mathrm{C}$ ?

Solution. Using the equation for converting ${ }^{\circ} \mathrm{C}$ to ${ }^{\circ} \mathrm{F}$ gives

$$
{ }^{\circ} \mathrm{F}=\frac{9}{5}(-4.5)+32=23.9^{\circ} \mathrm{F}
$$

Neither the Fahrenheit nor the Celsius scale provides an absolute standard because they arbitrarily assign numerical values to reference points, such as the boiling point and freezing point of water. The Kelvin scale, however, is an absolute temperature scale in which the lowest possible temperature is absolute zero, or 0 K . On the Celsius scale this is equivalent to $-273.15^{\circ} \mathrm{C}$; thus

$$
\mathrm{K}={ }^{\circ} \mathrm{C}+273.15
$$

Example 3. Report the temperatures in Examples 1 and 2 using the Kelvin scale.
Solution. For Example 1, the temperature is

$$
\mathrm{K}=38.1^{\circ} \mathrm{C}+273.15^{\circ} \mathrm{C}=311.25 \mathrm{~K}
$$

For Example 2, the temperature is

$$
\mathrm{K}=-4.5^{\circ} \mathrm{C}+273.15^{\circ} \mathrm{C}=268.65 \mathrm{~K}
$$

Volume. The SI unit for volume is the cubic meter $\left(\mathrm{m}^{3}\right)$, which is the volume of a cube whose sides each measure 1 m in length. This volume is much larger than that routinely used by chemists; thus, we will report volumes using the liter ( L ), where 1 L is equivalent to $1 \mathrm{dm}^{3}$ (a cube whose sides each measure 0.1 m or 10 cm ). Another common unit for volume is the milliliter ( mL ), where 1 mL is equivalent to $1 \mathrm{~cm}^{3}$ (a cube whose sides each measure 1 cm ). The prefix milli-, of course, means that 1 L contains 1000 mL .

Density. The density $(d)$ of an object is the ratio of its mass $(m)$ and its volume $(V)$; thus

$$
d=\frac{m}{V}
$$

Density is an example of an intensive property, by which we mean a property that doesn't depend on the amount of material. Consider, for example, a stick of chalk. If you break the chalk into two pieces of unequal size the density of the larger and smaller pieces are the same. The mass and volume of the larger piece, however, are greater than the mass
and volume of the smaller piece. Properties that depend on the amount of material, such as mass and volume, are called extensive properties.

Example 4. A piece of gold has a mass of 237 g and occupies a volume of $12.3 \mathrm{~cm}^{3}$. What is the density of gold?

Solution. The density of gold is

$$
d=\frac{237 \mathrm{~g}}{12.3 \mathrm{~cm}^{3}}=19.3 \mathrm{~g} / \mathrm{cm}^{3}
$$

## Scientific Notation

Many of the numbers with which chemists work are very large or very small. For example, in Module Two you will learn that a mole is a collection containing $602,200,000,000,000,000,000,000$ objects (this is called Avogadro's number). On the other hand, an electron weighs but 0.000000000000000000000000000910 grams. Obviously, expressing very large or very small numbers in decimal format is cumbersome.

Using Scientific Notation to Express Large and Small Numbers. Instead of using decimal notation to express large and small numbers, we will use scientific notation. In scientific notation we write numbers using a format of

$$
N \times 10^{n}
$$

where $N$ is a number between 1 and 10 , and $n$ is a positive or negative exponent indicating a power of 10 . To write a number in scientific notation we move the decimal point to the right or left until only one digit remains to the left of the decimal point (this is $N$ ). The number of spaces we move the decimal point is $n$, which is positive if we move the decimal point to the left and negative if we move it to the right.

Example 5. Express Avogadro's number and the mass of an electron using scientific notation.

Solution. Avogadro's number is $602,200,000,000,000,000,000,000$. Moving the decimal point 23 places to the left gives us $6.022 \times 10^{23}$.

An electron weighs 0.000000000000000000000000000910 grams. Moving the decimal point 28 places to the right gives us $9.10 \times 10^{-28} \mathrm{~g}$.

As shown in Table 3, the prefixes used in the SI system are best expressed using scientific notation.

Table 3. Prefixes Used in the SI System Showing Powers of 10

| Prefix | Symbol | Meaning | Prefix | Symbol | Meaning |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Tera- | T | $10^{12}$ | Centi- | c | $10^{-2}$ |
| Giga- | G | $10^{9}$ | Milli- | m | $10^{-3}$ |
| Mega- | M | $10^{6}$ | Micro- | $\mu$ | $10^{-6}$ |
| Kilo- | k | $10^{3}$ | Nano- | n | $10^{-9}$ |
| Deci- | d | $10^{-1}$ | Pico- | p | $10^{-12}$ |

A nanosecond, therefore, is conveniently expressed as $10^{-9} \mathrm{~s}$, and a kilometer as $10^{3} \mathrm{~m}$.
Scientific Notation and Calculators. When using a calculator to complete a calculation involving scientific notation, you need to be sure that you correctly enter the numbers; check your calculator's instruction manual if you are in doubt. Here is an example that you can use to verify that you are correctly entering numbers into your calculator.

Example 6. Complete the following calculation: $\left(5.3 \times 10^{6}\right) \times\left(9.1 \times 10^{-10}\right)$
Solution. Correctly entering the numbers into your calculator will give you an answer of

$$
\left(5.3 \times 10^{6}\right) \times\left(9.1 \times 10^{-10}\right)=4.823 \times 10^{-3}
$$

## Significant Figures

Consider the following rectangle. Find a ruler with a millimeter scale, measure the rectangle's width and length, and calculate its area, placing your results in the provided spaces.


$$
\begin{gathered}
\text { length }= \\
\text { width }= \\
\text { area }=
\end{gathered}
$$

How precisely did you make your measurements? Did you measure the length and width to the nearest mm , reporting values such as 48 mm and 28 mm , respectively? Or, did you estimate between the ruler's millimeter markings, reporting values such as 48.4 mm and 27.9 mm , respectively? For the area, did you report your answer to the nearest 10's place, the nearest 1's place, or to the nearest tenth or hundredth?

Uncertainty in Measurements. The questions in the preceding paragraph may seem unimportant (after all, why worry about the difference between reporting an area of 1344 $\mathrm{cm}^{2}$ vs. $1350 \mathrm{~cm}^{2}$ ). Nevertheless, these are crucial questions because they reflect a scientist's concern with precision. The last digit in a measurement is always an estimate and has some uncertainty. In measuring the length of the rectangle, I noted that its length was more than 48 mm . Estimating the length to be closer to 48 mm than 49 mm , I rounded my answer to 48 mm . When you look at this result, however, you can say only that the rectangle's true length must be between 47 mm and 49 mm , and that I either rounded up to 48 mm or rounded down to 48 mm . There is, therefore, an uncertainty in my measurement that is best expressed as $48 \mathrm{~mm} \pm 1 \mathrm{~mm}$, and a relative uncertainty of

$$
\frac{ \pm 1 \mathrm{~mm}}{48 \mathrm{~mm}} \times 100= \pm 2.1 \%
$$

By the same logic, reporting the length as 48.4 mm implies that the tenth's place is estimated with an absolute uncertainty of $\pm 0.1 \mathrm{~mm}$, and a relative uncertainty of

$$
\frac{ \pm 0.1 \mathrm{~mm}}{48.4 \mathrm{~mm}} \times 100= \pm 0.21 \%
$$

Estimating the length to the nearest 0.1 mm instead of the nearest millimeter decreases the percent uncertainty by a factor of 10 .

Example 7. Determine the relative uncertainty for a width of 28 mm . Repeat for a width of 27.9 mm .

Solution. Dividing the uncertainty of each measurement by its value gives the relative uncertainties as

$$
\frac{ \pm 1 \mathrm{~mm}}{28 \mathrm{~mm}} \times 100= \pm 3.6 \% \quad \frac{ \pm 0.1 \mathrm{~mm}}{27.9 \mathrm{~mm}} \times 100= \pm 0.36 \%
$$

Why this concern for uncertainty in measurements? Uncertainty becomes important because the result of a calculation cannot be more precise than the underlying measurements. Thus, if we measure the length and width of our rectangle to the nearest mm , its calculated area must have an uncertainty greater than $3.6 \%$ (the larger of the two relative uncertainties). If, on the other hand, we measure the rectangle's length and width to the nearest 0.1 mm , its calculated area should have an uncertainty of approximately $0.36 \%$. How does this compare to possible values for the area? Using a length of 48 mm and a width of 28 mm gives the area, $A$, as

$$
A=48 \mathrm{~mm} \times 28 \mathrm{~mm}=1344 \mathrm{~mm}^{2}
$$

If we report this value as the answer, then the relative uncertainty is

$$
\frac{ \pm 1 \mathrm{~mm}^{2}}{1344 \mathrm{~mm}^{2}} \times 100= \pm 0.074 \%
$$

which is significantly better than the minimum expected relative uncertainty of $3.6 \%$. Rounding the area to 1340 or 1300 gives relative uncertainties of, respectively

$$
\begin{aligned}
& \frac{ \pm 10 \mathrm{~mm}^{2}}{1340 \mathrm{~mm}^{2}} \times 100= \pm 0.75 \% \\
& \frac{ \pm 100 \mathrm{~mm}^{2}}{1300 \mathrm{~mm}^{2}} \times 100= \pm 7.7 \%
\end{aligned}
$$

The best answer, therefore, is $1300 \mathrm{~mm}^{2}$. If we take the length and width as 48.4 mm and 27.9 mm , respectively, then the calculated area is $1350 \mathrm{~mm}^{2} \pm 10 \mathrm{~mm}^{2}$, giving a relative uncertainty

$$
\frac{ \pm 10 \mathrm{~mm}^{2}}{1350 \mathrm{~mm}^{2}} \times 100= \pm 0.74 \%
$$

that is consistent with the minimum expected relative uncertainty of $0.36 \%$.
Determining the best answer for a calculation using the approach described above is tedious. In fact, we've significantly simplified the discussion and a more complete treatment of the "propagation of errors" is beyond the level of this course. Instead, we will rely on a few simple rules to guide us in properly reporting results. The rules aren't perfect, but most of the time they yield the correct result and we won't concern ourselves with the few instances where the rules fail.

Significant Digits and Significant Figures. What do we mean by the terms significant digit and significant figures? Basically, a significant digit is any number in a measurement in which we can express confidence. This includes all digits known exactly and the one digit (always the right-most digit) whose value is an estimate. For example, in measuring the width of the rectangle as 27.9 mm , the 2 and 7 are known exactly but the 9 is only an estimate. Each of these digits is significant, and the measurement of 27.9 mm is said to have three significant figures. The numbers 48 mm and $1344 \mathrm{~mm}^{2}$ have two and four significant digits, respectively.

When is a Zero a Significant Digit? We must exercise care when determining the number of significant figures in measurements containing one or more zeros. A zero is a significant digit only it represents an exact measurement of the one estimated digit; thus, the zeros in 2005 are significant and the number has four significant figures. A zero is never a significant digit if it serves merely as a placeholder to show the decimal point's location; thus, the zero in 0.055 are not significant and the number has two significant figures.

Example 8. How many significant figures are in the following measurements?

$$
503 \mathrm{~mm} \quad 0.160 \mathrm{~mm} \quad 150 \mathrm{~mm} \quad 0.0115 \mathrm{~mm}
$$

Solution. The measurement of 503 mm has 3 significant figures. Because the zero is between two significant digits, it must be a significant digit.

The measurement of 0.160 mm has 3 significant figures. Because the zero is the rightmost digit, it is the one digit whose value is an estimate and, therefore, it must be a significant digit. Note that the zero to the left of the decimal point is not significant as it only serves to locate the decimal point.

The measurement of 150 mm has 2 or 3 significant figures. In this example, the zero's role is ambiguous. It isn't clear if the measurement was estimated to the nearest $\pm 1 \mathrm{~mm}$, or if it was estimated to the nearest $\pm 10 \mathrm{~mm}$, with the zero serving as a placeholder to show the location of the decimal point (even though the decimal point isn't shown). To avoid confusion, it is best to express such numbers using scientific notation. A measurement of $1.5 \times 10^{2} \mathrm{~mm}$ has 2 significant figures and a measurement of $1.50 \times 10^{2}$ mm has 3 significant figures.

The measurement of 0.0115 mm has 3 significant figures (not 4!). The zeros in this example only serve to show the location of the decimal point. This becomes evident when the measurement is written in scientific notation as $1.15 \times 10^{-2} \mathrm{~mm}$.

Some numbers are exact and have an infinite number of significant figures. There are, for example, exactly 12 inches in a foot. When determining the average of three measurements

$$
\text { Exam Average }=\frac{97+86+92}{3}=92
$$

the number 3 is exact; thus we report the average to two significant figures, not one.
Calculations Using Significant Figures. As we learned when calculating the area of a rectangle, the number of significant figures must be consistent with the uncertainty in the measurements. As a general rule, mathematical operations involving addition and subtraction are carried out to the last place that is significant for all measurements included in the calculation. Thus, the sum of $135.621,0.33$, and 21.2163

| 135.621 |
| ---: |
| 0.33 |
| 21.2163 |
| 157.1673 |

is 157.17 since the last place that is significant for all three numbers (as shown by the vertical line) is the hundredth's place. Note that rounding to the correct number of significant figures occurs only after completing the exact calculation.

When multiplying and dividing, the general rule is that the answer contains the same number of significant figures as that measurement in the calculation having the fewest significant figures. Thus,

$$
\frac{22.91 \times 0 . \underline{152}}{16.302}=0.21361 \approx 0.214
$$

because 0.152 , with three, has the fewest significant figures.

$$
\frac{453-379}{112}=\frac{74}{112}=0.6607 \approx 0 . \underline{66}
$$

When a calculation includes multiple operations, apply the rules for significant figures at each step (but see discussion below about avoiding "round-off" errors). In this way you will report your final answer to the correct number of significant figures.

Avoiding "Round-Off" Errors. To avoid "round-off" errors in calculations it is a good idea to retain at least one extra significant figure throughout each step of the calculation. Better yet, invest in a good scientific calculator that allows you to perform lengthy calculations without recording intermediate values. When your calculation is complete, round the final answer to the correct number of significant figures using the following simple rules.

1. Retain the least significant figure if it and the digits that follow are less than half way to the next higher digit; thus, rounding 12.442 to the nearest tenth gives 12.4 since 0.442 is less than half way between 0.400 and 0.500 .
2. Increase the least significant figure by 1 if it and the digits that follow are more than half way to the next higher digit; thus, rounding 12.476 to the nearest tenth gives 12.5 since 0.476 is more than half way between 0.400 and 0.500 .
3. If the least significant figure and the digits that follow are exactly half way to the next higher digit then round the least significant figure to the nearest even number; thus, rounding 12.450 to the nearest tenth gives 12.4 , while rounding 12.550 to the nearest tenth gives 12.6. Rounding in this manner prevents us from introducing a bias by always rounding up or down.

Example 9. Complete the following calculations, reporting answers to the correct number of significant figures

$$
314-273.15=\frac{8.314 \times 298}{96485}=\quad \frac{0.250 \times 0.01123-0.100 \times 0.01927}{0.01123+0.01927}=
$$

Solution. For the first problem, when adding or subtracting the last decimal place common to both measurements determines the number of significant figures. As highlighted below, this is the one's place. The answer, therefore, is

$$
\underline{314}-\underline{273} .15=\underline{40.85 \approx 41}
$$

For the second problem, when multiplying or dividing the number of significant figures is determined by the measurement with the fewest significant figures. As highlighted below, the measurement with the fewest significant figures is 298, which has three significant figures. The answer, therefore, is

$$
\frac{8.314 \times \underline{298}}{96485}=\underline{0.0256783} \approx 0.0257=2.57 \times 10^{-2}
$$

For the third problem, when mixing together several calculations, complete each step of the calculation separately keeping at least one additional significant figure at each step (see highlighting below). Round the answer to the correct number of significant figures at the very end; thus

$$
\frac{\underline{0.250} \times 0.01123-\underline{0.100} \times 0.01927}{\underline{0.01123}+\underline{0.01927}}=\frac{\underline{0.002808}-\underline{0.001927}}{\underline{0.03050}}=\frac{\underline{0.000881}}{\underline{0.03050}}=\underline{0.0289} \approx 0.029
$$

## Dimensional Analysis

Much of the work you will do in this course involves converting a measurement with one unit into a result with a different unit. For example, in module 2 you will learn how to determine the number of moles of carbon dioxide, $\mathrm{CO}_{2}$, in 23.6 g of $\mathrm{CO}_{2}$, and in module 5 you will determine how many grams of $\mathrm{CO}_{2}$ are produced when burning 0.551 g of ethanol, $\mathrm{C}_{2} \mathrm{H}_{5} \mathrm{O}$. These calculations are not, from a mathematical perspective, very difficult. Nevertheless, it is easy to make simple mathematical mistakes, such as multiplying instead of dividing, when working with unfamiliar units.

Dimensional analysis, which is also called the factor-label method, is an approach to performing calculations that helps minimize errors. Consider this simple example - I am 6.0 feet tall. What is my in height inches? To convert between feet and inches I recall that 1 foot is equivalent to 12 inches, which I can express as one of two possible ratios

$$
\frac{1 \mathrm{ft}}{12 \mathrm{in}}=1 \quad \text { or } \quad \frac{12 \mathrm{in}}{1 \mathrm{ft}}=1
$$

As noted, both ratios have a value of 1 ; such ratios are often called unit conversion factors. To find the number of inches in 6.0 feet, I multiply 6.0 feet by the unit conversion factor that has inches in the numerator and feet in the denominator. Because I am multiplying by 1 , the thing I am measuring (my height) is unchanged except for the change in units.

$$
6.0 \mathrm{ft} \times \frac{12 \mathrm{in}}{1 \mathrm{ft}}=72 \mathrm{in}
$$

The answer, of course, has two significant figures. The advantage to keeping track of units in this way is that it prevents us from using the wrong unit conversion factor between feet and inches. Using the wrong unit conversion factor

$$
6.0 \mathrm{ft} \times \frac{1 \mathrm{ft}}{12 \mathrm{in}}=0.50 \mathrm{ft}^{2} \mathrm{in}^{-1}
$$

gives an answer that doesn't have the desired units (and that has no physical meaning!).

Example 10. A serving of plain M\&M's contains 27 g of sugar. How many pounds of sugar is this?

Solution. A pound (lb) is equivalent to 453.6 g . To convert between grams of sugar and pounds of sugar we use the unit conversion factor with grams in the numerator and pounds in the denominator; thus

$$
27 \mathrm{~g} \text { sugar } \times \frac{1 \mathrm{lb} \text { sugar }}{453.6 \mathrm{~g} \text { sugar }}=6.0 \times 10^{-2} \mathrm{lb} \text { sugar }
$$

Dimensional analysis also is useful for changing the prefix of a measurement reported in SI units.

Example 11. The World Health Organization recommends that the maximum allowable amount of arsenic in drinking water should be limited to $0.05 \mathrm{mg} / \mathrm{L}$. Express this as $\mu \mathrm{g} / \mathrm{L}$.

Solution. A microgram is $1 \times 10^{-6} \mathrm{~g}$ and a milligram is $1 \times 10^{-3} \mathrm{~g}$; thus

$$
\frac{0.05 \mathrm{mg}}{\mathrm{~L}} \times \frac{1 \times 10^{-3} \mathrm{~g}}{1 \mathrm{mg}} \times \frac{1 \mu \mathrm{~g}}{1 \times 10^{-6} \mathrm{~g}}=\frac{5.0 \times 10^{1} \mu \mathrm{~g}}{\mathrm{~L}}
$$

With practice you will find it easy to string together conversions between several units and to handle units that are raised to a power.

Example 12. An average adult has 5.2 L of blood. Express this volume in $\mathrm{m}^{3}$.
Solution. Three unit factors are needed for this problem, one to convert from liters to milliliters ( 1 L is 1000 mL ), a second to convert from milliliters to cubic centimeters ( 1 mL is $1 \mathrm{~cm}^{3}$ ), and a third to convert from cubic centimeters to cubic meters ( 1 m is 100 cm , thus $1^{3} \mathrm{~m}^{3}$ is $100^{3} \mathrm{~cm}^{3}$; note that both the number and the unit are cubed).

$$
5.2 \mathrm{~L} \times \frac{1000 \mathrm{~mL}}{\mathrm{~L}} \times \frac{1 \mathrm{~cm}^{3}}{1 \mathrm{~mL}} \times\left(\frac{1 \mathrm{~m}}{100 \mathrm{~cm}}\right)^{3}=5.2 \times 10^{-3} \mathrm{~m}^{3}
$$

## Practice Problems

The following problems provide practice in meeting this module's objectives. Answers are provided on the last page. Be sure to seek assistance if you experience difficulty with any of these problems. When you are ready, schedule an appointment for the module's exam.

Here are some useful relationships between units:
1 pound is 453.6 grams or 16 ounces
1 ton is 2000 pounds
1 mile is 1.609 kilometers or 5280 feet
1 inch is 2.54 centimeters
1 gallon is 3.7854 liters or 4 quarts
$1 \AA$ is $1 \times 10^{-8} \mathrm{~cm}$ or $1 \times 10^{-10} \mathrm{~m}$

1. Ethanol boils at $78.5^{\circ} \mathrm{C}$ and freezes at $-117^{\circ} \mathrm{C}$. Convert these temperatures to the Fahrenheit scale.
2. Mercury boils at $675^{\circ} \mathrm{F}$ and solidifies at $-38.0^{\circ} \mathrm{F}$. Convert these temperatures to the Celsius scale.
3. Convert the temperatures in problem 1 to the Kelvin scale.
4. Convert the temperatures in problem 2 to the Kelvin scale.
5. Calculate the density, in $\mathrm{g} / \mathrm{cm}^{3}$, of ethanol if $80.0 \mathrm{~cm}^{3}$ has a mass of 63.3 g .
6. Express the following numbers in scientific notation:
0.00000063

415
96485
0.0991
7. How many significant figures are there in the following measurements?
613.5 mi
30.6 mL
0.0000067 m

$$
3.50 \times 10^{8} \mathrm{~cm}
$$

8. Complete the following calculations, reporting your results to the correct number of significant figures.

$$
\begin{aligned}
& 0.0087+7.6 \times 10^{-3}= \\
& \frac{536}{4.2 \times 10^{2}}= \\
& \left(8.1 \times 10^{-5}\right) \times\left(4.8 \times 10^{6}\right)= \\
& \frac{3.60 \times\left(2.954 \times 10^{4}\right)}{\left(7.50 \times 10^{4}\right) \times(128.92-18.5+21)}=
\end{aligned}
$$

9. Express 3.69 m in kilometers, centimeters, and millimeters.
10. Express the mass of 32 g of oxygen in milligrams, kilograms, and pounds.
11. The color of light depends on its wavelength. The longest wavelength for visible light, which is at the red end of the spectrum, is $7.8 \times 10^{-7} \mathrm{~m}$. Express this length in micrometers, nanometers, and angstroms.
12. What is the volume, in liters, of a tank that is 0.6 m long, 10.0 cm wide, and 50 mm deep?
13. At Wimbledon, it isn't unusual to have tennis balls zipping around with a speed of $43 \mathrm{~m} / \mathrm{s}$. Express this speed in miles $/ \mathrm{hr}$.
14. Mercury has a density of $13.6 \mathrm{~g} / \mathrm{cm}^{3}$. What volume, in liters, will $3.00 \times 10^{2} \mathrm{~g}$ of mercury occupy?
15. An average person requires approximately 2.00 mg of riboflavin (vitamin $B_{2}$ ) each day. How many pounds of cheese would a person have to eat each day if this were his or her only source of riboflavin? Assume that each gram of cheese contains $5.5 \mu \mathrm{~g}$ of riboflavin.
16. Each gram of seawater contains $65 \mu \mathrm{~g}$ of bromine. Assuming that you are able to recover all the bromine from a sample of seawater, how many liters of seawater must you process to obtain 1.0 lb of bromine? Assume that the density of seawater is $1.0 \mathrm{~g} / \mathrm{mL}$.

## Answers to Practice Problems

1. $173^{\circ} \mathrm{F}$ and $-179^{\circ} \mathrm{F}$
2. $357^{\circ} \mathrm{C}$ and $-38.9^{\circ} \mathrm{C}$
3. 351.6 K and 156 K
4. $6.30 \times 10^{2} \mathrm{~K}$ and 234.2 K
5. $0.791 \mathrm{~g} / \mathrm{cm}^{3}$
6. $6.3 \times 10^{-7}$
$4.15 \times 10^{2}$
$9.6485 \times 10^{4}$
$9.91 \times 10^{-2}$
7. 4

3
2
3
8. 0.0163 or $1.63 \times 10^{-2}$
1.3
$3.9 \times 10^{2}$
$1.08 \times 10^{-2}$
9. $3.69 \times 10^{-3} \mathrm{~km}, 369 \mathrm{~cm}, 3.69 \times 10^{3} \mathrm{~mm}$
10. $3.2 \times 10^{4} \mathrm{mg}, 3.2 \times 10^{-2} \mathrm{~kg}, 7.1 \times 10^{-2} \mathrm{lb}$
11. $0.78 \mu \mathrm{~m}, 7.8 \times 10^{2} \mathrm{~nm}, 7.8 \times 10^{3} \AA$
12. 3 L

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13. 96 mph
14. $2.21 \times 10^{-2} \mathrm{~L}$
15. $0.80 \mathrm{lb} /$ day
16. $7.0 \times 10^{3} \mathrm{~L}$


[^0]:    ${ }^{\dagger}$ For example, at one time the absolute standard for mass was one cubic decimeter of water. Following the $1^{\text {st }}$ General Conference on Weights and Measures (held in 1889) the absolute standard was redefined as a specially manufactured rod of a platinum-iridium alloy stored at the International Bureau of Weights and Measures. This continues to be the absolute standard for mass.

[^1]:    $\dagger$ Many scientific calculators include a key that converts between ${ }^{\circ} \mathrm{F}$ and ${ }^{\circ} \mathrm{C}$; nevertheless, you should know how to make the conversion on your own.

