PV and IRR

You are teaching a child about money. You lay out three United States nickels, worth 5 cents each, in a row and then two dimes, each worth 10 cents, below the nickels, as in Figure 4.5.

Figure 4.5: Three Nickels and Two Dimes: Which is worth more?

You ask the child if the nickels or the dimes are worth more. The child says the nickels because there are more of them. Yes, more is better, you say, but nickels and dimes are not equivalent. How would you explain that two dimes are worth more than three nickels?

You could replace each dime with two nickels and now you have four nickels in the bottom row. With everything in nickels, we would easily see that the bottom row has more nickels and so it is worth more.

Or, if you have bunch of pennies lying around, you could replace each nickel with a stack of five pennies and each dime with a stack of ten pennies. With everything in pennies, we can use the child's "more is better" logic: 20 pennies is more than 15 pennies.

Both approaches rely on *standardizing* the two options so that we are comparing apples to apples. You have no problem knowing that two dimes are worth more than three nickels because you are instantly converting the coins to cents. You are standardizing without even knowing it.

If you think this is obvious, then you will have no problem with present value. It does the same thing, translating money in the future to money now so that we can correctly determine worth. Let's see how it works.

Present Value (PV)

Whenever you are faced with a decision involving money over time, there is a complication because of the fact that money has a *time dimension*. The confusing part is that we often ignore the time unit on money, but it is definitely there.

Everyone knows that $1,000 right now is worth way more than $1,000 fifty years from now. Both are denominated in dollars, but both also have a differing time unit so they are not equivalent.

Just like you cannot treat nickels and dimes as equivalent coins, you cannot directly compare numerical values of money at different points in time. You also would not say that "I have 3 cars and 4 pencils so I have 7 carpencils" because cars and pencils are different things and "carpencils" is meaningless. The same is true of money at different points in time.

If you had a portfolio of $1,000 in cash and a promise (that you could rely on with certainty) of $1,000 fifty years from now, you cannot say that this portfolio is worth $2,000. Although less obvious, this is the same as saying "I have 7 carpencils."

There is, however, a way to value your portfolio—convert the future money into present value. This is exactly the same as the strategy we used for the nickels and dimes problem in Figure 4.5.

The common time period chosen is usually the present and this is why it is called present value. Alternatively, future value uses a common time period in the future.

Suppose we want to know the present value of $10,000 five years from now with a 9% discount rate (dr). We are looking for the number that, if allowed to grow at 9% would be $10,000 five years from now.

STEP Open a blank Excel workbook and save it as *PVIRR.xlsx*. Enter the text *You get* in cell A1 and in cell B1, enter $10000 (using the $ automatically formats the cell as $). In cell C1, enter the text *in year* and enter the 5 in cell D1. In cell E1 enter the text *and dr is* and 9% in cell F1 (again, using % correctly formats the cell). Name cell F1 *dr* (the easiest way is by selecting cell F1 and then clicking in the *Name Box* and entering *dr*).

To be clear, present value is the amount needed right now (the present) so that it will grow into a given target value (in the future). Present value answers the question: "How much is a future amount of money worth today?"

STEP In cell A2 enter the label *Year* and in cell B2 enter the label *Value*. In cell A3, enter the number 5, followed by 4 in cell A4, 3 in cell A5, and so on down to 0 in cell A8. Enter the formula *=B1* in cell B3.

Now, what is the formula in cell B4? In previous work, we saw that the next number (t+1) in a geometric sequence was given by:

$$x_{t+1} = x_t + ix_t$$
$$x_{t+1} = (1+i)x_t$$

Here we are doing the reverse of that. Instead of the next number (which we know), we want the *previous* number that produced it.

$$x_{t+1} = (1+i)x_t$$
$$x_t = \frac{x_{t+1}}{(1+i)}$$

Instead of i, by convention, we use the phrase *discount rate* (*dr*) for the constant growth rate when we compute present value. We are discounting, or lowering, the number when we present value it. The % growth rate used is also sometimes abbreviated with the letter r. No matter what symbol you use, what you are doing is computing what the previous number in the sequence would be if it grew at that constant growth rate.

STEP In cell B4, enter the formula, *=B3/(1+dr)*, assuming you named cell F1, *dr*. Format cell B4 as $.

Excel shows $9,174.31. If you had that amount of money, it would grow to $10,000 in a year with a growth rate of 9% per year.

STEP Select cell B4 and fill it down to cell B8.

You are showing the present value of $10,000 five years from now with a 9% discount rate. In other words, if you had $6,499.31 right now, in five years it would become $10K by growing at 9% per year.

There is, of course, a faster way. We can bring a number for any future time period, t, back to the present, $t = 0$, with the *Present Value Equation*:

$$x_0 = \frac{x_t}{(1 + dr)^t}$$

The present value, x_0, depends on three inputs:

1. The amount in the future
2. The discount rate (which is the growth rate)
3. The number of time periods in the future

STEP In cell C8, enter the formula *=B1/(1+dr)ˆ5.*

The equivalence of cells B8 and C8 confirms that the one-step version of the present value computation works.

The Present Value Equation makes clear that farther in the future the lower the present value since you would divide by a bigger number as t rises. Similarly, the higher the discount rate the lower the present value.

STEP Change the discount rate in cell F1 to values higher and lower than 9% and keep track of the PV in cells B8 and C8.

If you set the discount rate to zero, then the present value is the same as the future amount. Without any growth rate, to reach the target future amount, you have to start with that future amount.

The Lottery Decision

The biggest lottery jackpot as of this writing was on November 8, 2022, to one Powerball ticket in California for $2.08 billion! Lotteries always advertise their jackpot as the total payout even though it is distributed over time.

As usual, the winner took the immediate cash alternative, which was $997.6 million—a little less than half of what they would have received in 30 payments over 29 years. There are many ways to think about how to decide between taking a single amount now versus a stream of payments. One way is to compute the present value of the stream of payments and compare it to the cash alternative.

Real-world lotteries have complicated payout schemes with rising amounts over time so we will consider a simple, hypothetical lottery with a constant payout. Let's suppose that the jackpot is $1.5 million which is paid out in 30 payments of $50,000 each year. The immediate cash alternative is $750,000.

So, which one would you take and why? We agree that saying, "Give me the $1.5 million because it is more than $750,000" is exactly equivalent to the child saying, "I like 3 nickels more than 2 dimes because there are more of them." We have to do the present value math. If you are leaning toward the lump sum now, present valuing might change your mind.

STEP Insert a new sheet in your *PVIRR.xlsx* workbook. Enter the label *Year* in cell A1 and the label *Amount* in cell B1. In cell A2 a 0 followed by a 1 in cell A3. Select cells A2 and A3 and then fill down to cell A31. It should show a value of 29. Enter $50000 in cell B2 and the formula =B2 in cell B3. Fill it down. This way we can easily change the payment stream. Finally, sum the $50,000 payments in cell B32.

Of course, cell B32 should show $1.5 million. Less obvious is the fact that in a real sense, this is an illegal sum. The $50,000 come at different times so they are not equivalent. Excel (and the lottery people) do not care about the time dimension of money, but ignoring the time unit does not make it go away.

We can compute the sum correctly if we present value all of the payments. That way, we will be adding dollars that have the same time unit—the present.

STEP In cell C1 enter the label *PV* and in cell D1 enter 5%. Name cell D1 *IRR* (explained below). In cell C2 enter the formula *=B2/(1+IRR)^A2* and fill it down.

Cell C2 is the same as B2 because the present value of any amount right now is that amount. The zero in the exponent for time makes the denominator 1. But look at the other values in column C. They are getting smaller and smaller as you look down. That is because it takes less money now (present value) to grow to $50,000 the farther in the future we get the $50,000.

STEP Copy cell B32 and paste it in cell C32.

You are looking at the correct way to sum the stream of payments. We have a single number, $807,054, to represent its value. The "simple" addition of the 30 payments of $50,000 is a gross misrepresentation of the true value of the stream of payments. It is amazing to people who understand the time-value of money that lottery jackpots are allowed to be advertised as the sum of payments over time.

We would never compare the $1.5 million to the immediate cash payout, but we can directly compare the present value to the immediate cash payout. With 5% per year, the present value of the stream of payments is greater than the immediate payout so we would take the stream of payments.

There is one problem, however, the present value depends on the discount rate used.

STEP Change cell D1 to 10% and scroll down to see the present value of the stream of payments.

The future amounts are all smaller (since the growth rate is bigger) and the sum of the present values for each payment is $518,480. Since this is less than the immediate cash payout of $750,000, we should take the cash amount in this case.

This shows that there is no one-size-fits-all answer to deciding the question of cash or stream of payments. It depends on the discount rate used. If you had the ability to invest the immediate cash payout at 10% per year, the cash payout is worth more to you. If, however, the best you can do is 5% per year, then you would take the stream of payments.

Internal Rate of Return (IRR)

We can use the fact that the present value depends on the discount rate to explain the internal rate of return. We begin by computing the IRR (also known as the ROR or rate of return) for our hypothetical lottery question of how to take our winnings.

STEP In cell F1 enter the label *Amount* and in cell F2 enter \$750000. Enter \$0 in cell F3 and then fill it down to cell F31.

Columns B and F now have two alternative streams. One is \$50,000 from now until year 29 and the other is \$750,000 now and nothing until year 29. We subtract column F from A to create a single series that captures two streams.

STEP In cell H1 enter the label *Net Amount* and in cell H2 enter the formula *=B2-F2*. Fill it down to cell H31.

Column H makes clear that we can frame the question of how to take our lottery winnings as an investment project. Cell H2 shows negative \$700,000 (the parentheses and red text signal that the dollar amount is less than zero) so this is what we are committing to this project. In return, we get the stream of \$50,000 payments in the future.

We need to present value the amounts in column H in order to make them have the same time unit, the present.

STEP In cell I1 enter the label *NPV* and in cell I2 enter the formula *=H2/(1+IRR)^A2*. Fill it down to I31. Copy cell C32 and paste it in cell I32.

Cell I32 shows the net present value for a given discount rate. If this number is positive, then the investment project (\$50,000 over 29 years) is worthwhile; if it is negative, it is not. At 5%, the project is worthwhile (take the stream of payments); at 10% it is not (take the immediate cash payout).

This work leads us to an interesting question: what is the break-even discount rate? In other words, what is the percentage growth rate that would make the net present value be zero?

STEP In cell K1 enter the label *IRR* and in cell K2 enter the formula *=IRR(H2:H31)*. You do not need to provide a *guess* parameter.

The Excel function IRR computes the internal rate of return which is the 5.72% displayed in cell K2. The IRR function is solving for the value of IRR in this equation:

$$0 = \frac{x_0}{(1+IRR)^0} + \frac{x_1}{(1+IRR)^1} + \frac{x_2}{(1+IRR)^2} + ... + \frac{x_{29}}{(1+IRR)^{29}}$$

There is no analytical solution for finding the value of IRR in the equation above so Excel uses an iterative algorithm. The guess parameter helps the function find a solution in more complicated investment projects. In fact, for really complicated projects with costs and returns appearing over time, the IRR method can fail. Such projects require use of the NPV method.

STEP Copy cell K2, select cell D1, click the down arrow on the *Paste* item in the *Ribbon* (in the *Home* tab) and paste *Values*.

Cell I32 is now displaying a value that is almost zero. "E-10" means 10 to the minus 10 power, so it has ten zeroes after the decimal point. This demonstrates that Excel's IRR function correctly computed the IRR, the value of the discount rate that sets the net present value to zero.

The IRR tells us the quality of the investment project. The higher the IRR the better the project.

We can compare the IRR of different projects. Suppose you have an investment opportunity that earns you 10%. Then you would take the immediate cash payout. However, if your best investment option yields only 5%, then the IRR of this project is higher and you would take the stream of payments.

Notice that we get the same answer to which option to take if we use the discount rate to compute the present value of the stream or compare that same discount rate to the IRR.

Takeaways

Suppose someone asked you if you'd prefer 100 US dollars or 840 Tajikistani somoni (yes, that is the currency of Tajikistan). Which would you choose? You certainly would not say, "840 is more than 100 so I will take the somoni."

You cannot answer until you use the exchange rate to convert the 840 somoni to dollars or 100 dollars to somoni. Once they are in the same units, you can compare the values.

Just as you need an exchange rate to compare money denominated in different currencies, you need to convert money paid or received at different points in time to a common denominator so that you can make the right comparison.

We say, "The present value is $100" to express the value of a future amount today.

We also use present value as a verb, "to present value" means to do the computation of dividing the future amount by $(1 + dr)^t$.

The *Present Value Equation* is:

$$x_0 = \frac{x_t}{(1 + dr)^t}$$

Choosing the discount rate can be complicated and is beyond the scope of this book. It depends on the riskiness of the investment and other factors.

Especially for projects with long time horizons, present value can be extremely sensitive to the chosen discount rate.

The higher inflation the more discounting of a future amount of money, but the concept of present value does not depend on rising price levels. Even with zero inflation (constant prices) we would see positive discount rates and, therefore, discounting of money in the future.

The IRR is the discount rate that sets the NPV = 0.

The IRR is a number that can be used to judge an investment project. If $IRR > hurdlerate$ (the market interest rate or cost of funds), then the project is worth pursing.

The IRR only works for simple investment projects with upfront costs and future returns. For more complicated projects, the NPV method should be used.

The NPV method uses the decision-maker's discount rate (the market interest rate or cost of funds) to present value the net stream. If $NPV > 0$, then the project is worth pursuing.

The NPV and IRR methods will give the same answer. Thus, using both methods is a good check on the computations and final decision.

Lottery jackpots are misleading because they do not advertise the immediate cash option. They announce total dollars over time which is like saying that the winner gets 7 carpencils.

Most people choose the immediate cash payout, but this decision should include consideration of the time value of money.