

Comparative Statics

Your doctor is some kind of evil genius (or an economist) and she wants to explore your response to different values of the total allowed units of cigars and brandies.

This is formally known as *comparative statics analysis*. We change one exogenous variable and we see what effect it has on the optimal solution.

There's an Excel add-in for that—it is called the Comparative Statics Wizard (CSWiz). After explaining comparative statics in more detail, we will use the CSWiz add-in on the constrained optimization version of the cigars and brandies problem.

The Logic of Comparative Statics

We did comparative statics analysis earlier with the Lifeguard Problem. We turned the lifeguard into a freakish combination of Michael Phelps and Usain Bolt. The optimal solution had us running more since we were so much faster.

We also did comparative statics analysis when we changed the product price facing the farmer. As product price rises, the farmer responds by buying more seed.

A third example involved pooled testing. We lowered the infection rate from 5% to 1% and this made the optimal group size bigger.

All of these examples worked the same way. We solved an optimization problem, then we changed a single exogenous variable and kept everything else the same. We focused on the effect the change had on the optimal solution.

Before we do another example, let's repeat and make crystal clear the logic of comparative statics. It involves a four-step procedure.

1. We set up the problem and find the *initial* solution.
2. We change a single exogenous variable, called the *shock*, holding all other exogenous variables constant. We use a Latin phrase, *ceteris paribus*, as shorthand. This literally means *with other things held equal* and we use the phrase to mean *everything else held constant*.
3. We find the *new* optimal solution.
4. Finally, we *compare* the new to the initial solution to see how the optimal solution responded to the shock.

Comparative statics is the fundamental methodology of economics. It gives a framework for interpreting observed behavior. This framework has been given many names, including: the method of economics, the economic approach, the economic way of thinking, and economic reasoning.

As you know, *comparative* clearly points to the comparison between the new and initial solution, the meaning of *statics* (not be confused with statistics) is less obvious. It means that we are going to focus on positions of rest and not worry about the path of the solution as it moves from the initial to the new point.

We have several ways of comparing the new and initial solutions. A qualitative comparison focuses only on direction (up or down), while quantitative comparisons compute magnitudes of the change in response (either as a difference or a percentage change).

Another Example

We start with the initial constrained optimization problem, which can be formally written like this:

$$\begin{aligned} \max_{B,C} U &= 18B - 3B^2 + 20C - C^2 \\ \text{s.t. } B + C &\leq 5 \end{aligned}$$

We know that the initial solution is $B^* = 1$ and $C^* = 4$ and $U^* = 79$.

Your doctor surprises you at your next appointment. She is happy with your latest test results and says that you can safely have one more brandy or cigar, so your total allowed amount is now 6. How will you respond to this new development?

STEP From your *ConOpt* sheet in your *UtilityMax.xlsx* workbook, change the 5 in cell A6 to 6. What happens?

The constraint cell is now negative. This means that you are below the constraint. The barrier has moved upwards so you have room to climb higher up the utility hill. But what exactly will you do?

STEP Run Solver. What happens?

With a total of 6 cigars and brandies allowed, your new optimal solution is $B^* = 1.25$ and $C^* = 4.75$ yielding a maximum utility of $U^* = 90.25$

This makes sense. You take advantage of the loosened constraint to sip a little more brandy and smoke more cigars. That is a qualitative or directional statement. It is like saying that when the price goes down, you buy more.

A quantitative or magnitude statement would be to compute how much more you drink and smoke. You went from 1 to 1.25 brandies as the total amount allowed went from 5 to 6 so that increase is 0.25 brandies. The delta (or difference) for cigars is 0.75 since you went from 4 to 4.75.

There is another way to make a quantitative statement using elasticity. The Comparative Statics Wizard and the results it generates will help us explain how to compute and interpret an elasticity.

The Comparative Statics Wizard

We use a free Excel add-in, *CSWiz.xla*, to do comparative statics analysis. It works with Solver to find the optimal solution given different values of an exogenous variable.

STEP Download the *CSWiz.xla* file from tiny.cc/addin and use the Add-Ins Manager (File → Options → Add-ins → Go or keyboard shortcut *alt, t, i*) to install it. Once installed, click the *Add-ins* tab to see that it is under the *Wizard* group.

To use the CSWiz add-in, we need to modify our Excel implementation of the constrained optimization problem. Instead of hard-coding the total allowed amount of cigars and brandies, we need to make a cell for this exogenous variable.

STEP In cell B8 of your *ConOpt* sheet, enter the label *Total*. In cell A8, enter the number 5. Connect this total value to the constraint cell in A6 by changing the constraint formula to $= A1 + A2 - A8$.

We are now ready to run the Comparative Statics Wizard. We will provide information in a series of steps.

STEP Click the *Add-ins* tab, click *Wizard*, and then *Comp Statics*.

As you walk through the steps, be sure to read carefully and think about the information you are providing. Below is the input you need to give as you walk through the steps.

1. Clicking the button produces an input box that asks for the objective function cell. Click on cell A4 and click *OK*. A second box asks for the endogenous variables. Select cells A1 and A2 (both of them) and click *OK*. The final input box asks for the exogenous variables. Click on cell A8 and click *OK*. Usually, there are more than one exogenous variables so you would select all of them. When done, you return to the Wizard dialog box, but your inputs are displayed. Confirm that they are correct and click *Next*.
2. Click the button to call Solver and run it. Solver needs to successfully find the correct solution to continue. If not, we cannot do comparative statics analysis. When done, you return to the Wizard dialog box. Click *Next*.
3. This step is like the first one in that you are asked three questions. Click the button. Click cell A8 because this is the cell that we want to vary to see how the optimal solution responds. Click *OK*. In the second input box, enter the number 1. This will change cell A8 by 1. Click *OK*. In the final input box, leave the default choice of 5. Click *OK*. You return to the Wizard and the results of your input are displayed. Confirm that everything is correct and click *Next*.

4. The Wizard now has all of the information it needs. Clicking the Run Comparative Statics Analysis button will do just that. Excel will solve the problem for total values from 5 to 10 by 1. The *Progress Bar* will advance quickly because this problem is simple and we only asked for 5 shocks. Click *Next*.
5. Read the message on the final screen and click *Finish*.

You are taken to a new worksheet in your UtilityMax.xlsx workbook that displays the results of the comparative statics analysis that you just performed. We use these results to figure out and explain how changing the total allowed amount affects the optimal consumption of brandies and cigars.

STEP We did not name any cells so the results show cell addresses. We can fix this by entering the names of the variables. Cells A5 and A8 are *Total*. Cells B8, C8, and D8 are $Utility^*$, B^* , and C^* , respectively. Widen the columns if needed to display the results neatly.

The results confirm our work for $Total = 5$ and 6, but it extends the comparative statics analysis to total allowed amounts of 7, 8, 9, and 10. Of course, Solver's numbers suffer from false precision, but we know how to interpret them.

STEP Enter the text $DB/DTotal$ in cell E8. In the formula bar, select the first D and change the font to Symbol (in the *Home* tab). Repeat for the second D.

EXCEL TIP Excel allows you to apply formatting to individual characters in a cell. This means you can use different fonts, colors, and sizes for different parts of a cell.

STEP In cell E10, enter the formula $=(C10 - C9)/(A10 - A9)$ and fill it down.

That is interesting—every time you get an extra total amount allowed, you devote 0.25 of it to brandies (and 3/4 to cigars). In other words, the relationship between optimal brandies and the total amount allowed is linear. We can confirm this with a graph.

STEP Make a chart of B^* as a function of *Total*.

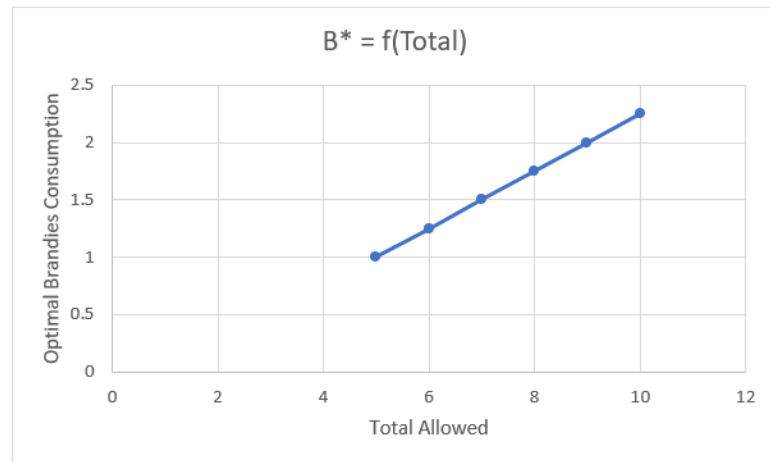


Figure 6.10: How brandies consumption responds to total allowed.

Your chart should look like Figure 6.10. Notice that we are not displaying an optimal solution. Instead, this chart is tracking how the optimal solution responds to changes in an exogenous variable. This is a common way to present the results of comparative statics analysis.

There is a similar chart of optimal cigars. The relationship is linear, but the slope is bigger so the line is steeper. It would be good practice to compute the slope and make this chart.

Takeaways

People often think that economics is defined by its content. They think that if you study something like unemployment or money, then you are doing economics.

Actually, modern economics is defined by its methodology. Economists use optimization and comparative statics on anything involving choice. Economics can be applied to war, marriage, and many other “non-economic” questions.

You can be sure that optimization and comparative statics will be applied if you ever see a title that begins with “An Economic Analysis of”—this means that whatever is being studied will be analyzed and seen as an optimization problem.

We use comparative statics to interpret changes in behavior (you sipped more brandy when the constraint loosened) and to predict responses (lowering the price will trigger an increase in quantity demanded).

The Comparative Statics Wizard is a numerical approach, as opposed to analytical approaches that use mathematics.

CSWiz.xla is an Excel add-in that takes advantage of Excel's Solver add-in. The user provides information about the problem and which variable is to be shocked. CSWiz does the tedious work of solving the problem at different values of the exogenous variable and keeps track of the optimal solutions.

Once you have the results, further analysis can be performed. Often, we are interested in the relationship between variables and we draw graphs to visualize how an endogenous variable responds to an exogenous shock.