# Elasticity

You probably have heard of the price elasticity of demand, but you may not know what it means or how to use the concept.

Our goal is to truly understand elasticity and be comfortable using it. At its most fundamental level, it is simply a numerical measure of responsiveness.

In the initial version of the Lifeguard Problem, the lifeguard entered the water after running roughly 56 meters. When maximum running speed doubled to 10 m/sec, ceteris paribus, the optimal solution changed to running almost 80 meters. We can (and did) show this on a graph, but is there a faster way to summarize comparative statics analysis? Yes, in a word, *elasticity*.

We have been working on a constrained optimization problem where you sipped one brandy and smoked four cigars when the doctor set a limit of 5 total brandies and cigars. When that limit was relaxed and you were allowed a total of 6 units, you chose 1.25 brandies and 4.75 cigars. Again, we can (and did) show this on a graph, but elasticity captures the relationship between the amount consumed and the total allowed in a single number.

We proceed by reviewing a few general ideas about elasticity and what it is trying to convey. Then we move to actual computations and practice interpreting elasticity values.

The more examples you see, the more the concept will stick. Be sure to keep an eye out for the repeated pattern in the elasticity. We always have an optimal solution that is responding to a shock and the elasticity measures if the response is weak or strong.

## Elasticity Basics

*Elasticity* is a pure number (it has no units) that measures the sensitivity or responsiveness of one variable when another changes. Elasticity, responsiveness, and sensitivity are synonyms. An elasticity number expresses the impact one variable has on another. The closer the elasticity is to zero, the more insensitive or inelastic the relationship is between two variables.

Elasticity is often expressed as "the something elasticity of something," like the price elasticity of demand. The first something, the price, is always the exogenous variable; the second something, in this case demand (the amount purchased), is the response or optimal value being tracked.

A less common, but perhaps clearer, way to express the cause and effect is to say, "the elasticity of something with respect to something." The elasticity of demand with respect to price makes it clear that demand depends on and responds to the price.

Unlike the difference between the new and initial values, elasticity is computed as the ratio of percentage changes in the values. The endogenous or response variable always goes in the numerator and the exogenous or shock variable is always in the denominator. Thus, the $x$ elasticity of $y$ is $\frac{\%\Delta y}{\%\Delta x}$.

The percentage change, $\frac{new - initial}{initial}$, is the change (or difference), new minus initial, divided by the initial value. This affects the units in the computation. The units in the numerator and denominator of the percentage change cancel and we are left with a percent as the units. If we compute the percentage change in apples from 2 to 3 apples, we get a 50% in increase. The change (or delta), however, is +1 apple.

If we divide one percentage change by another, as we do with an elasticity computation, $\frac{\%\Delta y}{\%\Delta x}$, the percents cancel and we get a unitless number. Thus, elasticity is a pure number with no units. So if the price elasticity of demand for apples is $-1.2$, there are no apples, dollars, percents, or any other units. It's just $-1.2$.

The $-1.2$ can be used to compute the percentage change in apples if the price of apples increases by 10%. We simply multiply $-1.2$ by 10% to get $-12\%$. Or, if the price of apples falls by 20%, we know that the quantity demanded of apples will rise by 24% ($-1.2 \times -20\%$).

We can also use an elasticity to compute the exogenous shock needed to produce a given percentage change in the endogenous variable. If ApplesRUs, Inc knew that the price elasticity of demand for apples was $-1.2$ and they wanted to increase apples sold by 6%, then they would lower prices by 5% (6% divided by $-1.2$ ).

Elasticity is a ratio of percentages changes so there are three numbers involved: the elasticity, the percentage change in the numerator, and the percentage change in the denominator. We are given and use two of the three to find the third one:

1. Given $\%\Delta x$ and $\%\Delta y$, find the elasticity: $\frac{\%\Delta y}{\%\Delta x}$.
2. Given $\%\Delta x$ and elasticity, find the $\%\Delta y$ : $elasticity \times \%\Delta x$.
3. Given $\%\Delta y$ and elasticity, find the $\%\Delta x$: $\frac{\%\Delta y}{elasticity}$.

The lack of units in an elasticity measure means we can compare wildly different things. No matter the underlying units of the variables, we can put the dimensionless elasticity number on a common yardstick and interpret it.

Figure 6.11 shows the possible values that an elasticity can take, along with the names we give particular values.

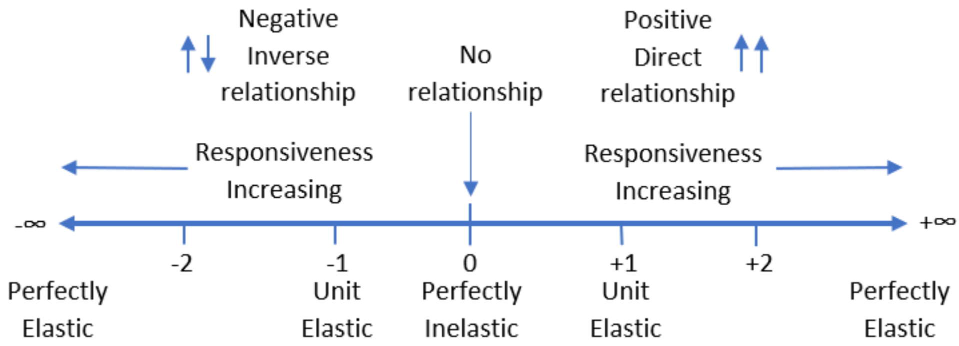


Figure 6.11: Elasticity on the number line.

Empirically, elasticities are usually low numbers around one (in absolute value). An elasticity of $+2$ is extremely responsive or elastic because the response is twice the shock. It means that a 1% increase in the exogenous variable generates a 2% increase in the endogenous variable.

The sign of the elasticity indicates direction (a qualitative statement about the relationship between the two variables). Zero means that there is no relationship—i.e., that the exogenous variable does not influence the response variable at all. Thus, $-2$ is extremely responsive like $+2$, but the variables are inversely related so a 1% *increase* in the exogenous variable leads to a 2% *decrease* in the endogenous variable.

One (both positive and negative) is an important marker on the elasticity number line because it tells you if the given percentage change in an exogenous variable results in a smaller percentage change (when the elasticity is less than one), an equal percentage change (elasticity equal to one), or greater percentage change (elasticity greater than one) in the endogenous variable.

The adjective *perfectly* is used to identify two extreme cases. If the elasticity is zero, it is perfectly inelastic and this means there is no response at all to a shock. This is rare, usually optimal values of endogenous variables adjusts to changes in the environment.

Perfectly elastic means the elasticity measure is infinity (positive or negative). This means that the tiniest little change in an exogenous variable triggers a massive response in the endogenous variable. Again, this is a rare, limiting case for elasticity.

Elasticities can be confusing. There is a lot to remember. Below are six common misconceptions and issues surrounding elasticity. Reading these typical mistakes will help you better understand this fundamental, but easily misinterpreted, concept.

1. Elasticity is about the *relationship* between two variables, not just the change in one variable. Thus, do not confuse a negative elasticity as meaning that the response variable must decrease. The negative means that the two variables move in opposite directions. So, if the age elasticity of time playing sports is negative, that means both that time playing sports falls as age increases and time playing sports rises as age decreases.

2. Elasticity is a *local phenomenon*. The elasticity will usually change if we analyze a different initial value of the exogenous variable. Thus, any one measure of elasticity is a local or point value that applies only to the change in the exogenous variable under consideration from that

starting point. You should not think of a price elasticity of demand of $-0.6$ as applying to an entire demand curve. Instead, it is a statement about the movement in price from one value to another value close by, say \$3.00/unit to \$3.01/unit. The price elasticity of demand from \$4.00/unit to \$4.01/unit may be different. There are constant elasticity functions, where the elasticity is the same all along the function, but they are a special case.

3. Elasticity can be calculated for *different size changes.* To compute the $x$ elasticity of $y$, we can go from one point to another, $\frac{\%\Delta y}{\%\Delta x}$, but the size of the change in $x$ can vary. The computed elasticity will be different depending on the size of the shock if the relationship is non-linear.

4. Elasticity always puts the *response variable in the numerator.* Do not confuse the numerator and denominator in the computation. In the $x$ elasticity of $y$, $x$ is the exogenous or shock variable and $y$ is the endogenous or response variable. Students will often compute the reciprocal of the correct elasticity. Avoid this common mistake by always checking to make sure that the variable in the numerator responds or is driven by the variable in the denominator.

5. Remember that elasticity is *unitless.* The $x$ elasticity of $y$ of 0.2 is not 20%. It is 0.2. It means that a 1% increase in $x$ leads to a 0.2% increase in $y$.

6. Perhaps the single most confusing thing about elasticity is its relationship to the slope: *Do not confuse elasticity with slope.* This is easy to forget and deserves careful consideration. Remember that elasticity is a ***percentage*** change calculation, $\frac{\%\Delta y}{\%\Delta x}$, while a slope is merely the rise over the run, $\frac{\Delta y}{\Delta x}$.

Economists, unlike chemists or physicists, often gloss over the units of variables and results. If we carefully consider the units involved, we can ensure that the difference between the slope and elasticity is crystal clear.

The slope is a quantitative measure in the units of the two variables being compared. If $Q^* = \frac{P}{2}$, then the slope, $\frac{\Delta Q^*}{\Delta P} = \frac{1}{2}$. This says that an increase in P of \$1/unit will lead to an increase in $Q^*$ of $\frac{1}{2}$ a unit. Thus, the slope would be measured in units squared per dollar (so that when multiplied by the price, we end up with just units of Q).

Elasticity, on the other hand, is a quantitative measure based on percentage changes and is, therefore, unitless. The $P$ elasticity of $Q* = 1$ says that a 1% increase in $P$ leads to a 1% increase in $Q*$. It does not say anything about the actual, numerical \$/unit increase in $P$, but speaks of the percentage increase in $P$. Elasticity focuses on the percentage change in $Q*$, not the change in terms of number of units.

Thus, elasticity and slope are two different ways to measure the responsiveness of a variable as another variable changes. Elasticity uses percentage changes, $\frac{\%\Delta y}{\%\Delta x}$, while the slope does not, $\frac{\Delta y}{\Delta x}$. They are two different ways to measure the effect of a shock and confusing them is a common mistake.

# Computing Elasticity

When the total allowed B and C went from 5 to 6, you changed B$^*$ from 1 to 1.25 and C$^*$ from 4 to 4.75. We can compute two elasticities with these numbers.

The total allowed elasticity of brandies is

$$\frac{\%\Delta B*}{\%\Delta T} = \frac{\frac{\Delta B^*}{B^*}}{\frac{\Delta T}{T}} = \frac{\frac{newB - initialB}{initialB}}{\frac{newT - initialT}{initialT}} = \frac{\frac{1.25-1}{1}}{\frac{6-5}{5}} = \frac{0.25}{0.2} = 1.25$$

The total allowed elasticity of brandies is 1.25 because we had a 20% increase in T (from 5 to 6) and this led to a slightly bigger, 25% increase in brandies (from 1 to 1.25). Thus, we say that the brandies response is elastic, or pretty responsive. Figure 6.11 shows that any elasticity greater than 1 in absolute value is said to be elastic.

The total allowed elasticity of cigars is

$$\frac{\%\Delta C*}{\%\Delta T} = \frac{\frac{\Delta C^*}{C^*}}{\frac{\Delta T}{T}} = \frac{\frac{newC - initialC}{initialC}}{\frac{newT - initialT}{initialT}} = \frac{\frac{4.75-4}{4}}{\frac{6-5}{5}} = \frac{0.1875}{0.2} = 0.9375$$

The total allowed elasticity of cigars is 0.9375 because we had a 20% increase in T (from 5 to 6) and this led to a slightly smaller, 18.75% increase in cigars (from 4 to 4.75). Thus, we say that the cigars response is inelastic, or unresponsive. Figure 6.11 shows that any elasticity less than 1 in absolute value is said to be inelastic.

Since we know the slope of the optimal brandies as a function of total allowed is 0.25 and the total allowed elasticity of brandies is 1.25, that is conclusive proof that elasticity and slope are different. Another way we can show the difference is with a little algebra.

$$\frac{\%\Delta B*}{\%\Delta T} = \frac{\frac{\Delta B*}{B*}}{\frac{\Delta T}{T}} = \frac{\Delta B*}{B*}\frac{T}{\Delta T} = \frac{\Delta B*}{\Delta T}\frac{T}{B*}$$

The last term shows we can compute the elasticity by multiplying the slope by $\frac{T}{B*}$. This shows that elasticity is slope times the ratio of the exogenous to the endogenous variable values. In this example, 0.25 times 5/1 is 1.25.

We can also show that elasticity changes as you change the point from which it is measured.

*STEP* In your *CS1* sheet, put the label *%DB/%DT* in cell F8 and then change the two *D*s to Symbol font. In cell F10, enter the formula *=((C10 - C9)/C9)/((A10 - A9)/A9)* and fill it down.

Cell F10 reproduces the 1.25 elasticity we computed earlier, but notice how the elasticities get smaller as $T$ rises. Again, this shows that elasticity is not slope since the slope stays constant while the elasticity changes.

## Elasticity Practice

Work on these elasticity computations and questions to improve your understanding. Answers are provided in the appendix (according to step number).

*STEP* 1. Compute the max run speed elasticity of distance on sand in the lifeguard problem as max run speed is increased from 5 m/sec to 10 m/sec. Interpret your result.

*STEP* 2. Compute the IR (infection rate) elasticity of group size as IR falls from 5% to 2%. Recall that optimal group size rose from 5 to 8. Interpret your result.

*STEP* 3. Compute the slope of $C* = f(T)$ and use it to compute the total allowed elasticity of cigars at T = 5. Does your number agree with the 0.9375 value we found earlier?

*STEP* 4. Compute the slope of $C^* = f(T)$ and the total allowed elasticity of cigars from T = 9 to 10. Does the slope or elasticity change compared to the elasticity from T = 5 to 6? What does this show?

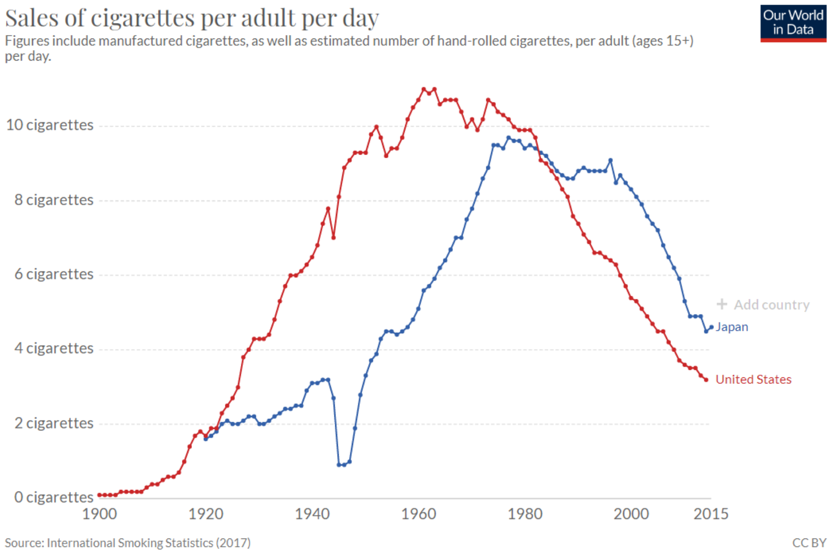


Figure 6.12: Smoking rates in Japan and the United States. *Source: ourworldindata.org/smoking*

Cigarettes have been extensively studied. The average number of cigarettes sold per day in the United States and Japan since 1900 is shown in Figure 6.12.

Visit ourworldindata.org/smoking to see an interactive version of this chart and to add other countries. The pattern is the same around the world—rising smoking rates reach a peak, then they decline. Today, a little over 10% of American adults smoke, down from 40% at the peak.

Governments want to reduce cigarette consumption to improve public health. Banning advertising is common, as is taxing cigarettes. The idea is that increasing the total price consumers must pay (the price plus the tax) will reduce consumption. Whether this works depends on the price elasticity of demand (the quantity purchased).

*STEP* 5. To reduce cigarette consumption in response to a tax, what are governments hoping is true about the price elasticity of demand for cigarettes?

*STEP* 6. What do you think is a good guess for the price elasticity of demand for cigarettes? Explain your answer.

*STEP* 7. How do you think the price elasticity of demand for cigarettes compares between adult and teenage smokers? Explain your answer.

## Takeaways

Comparative statics is how economists view the world and elasticity is how they communicate comparative statics results.

You want to be able to interpret and compute it:

Interpret: The closer an elasticity is to zero, the less responsive the endogenous variable is to a particular shock.

Compute: The exogenous variable elasticity of the endogenous variable is always the percentage change in the endogenous variable divided by the percentage change in the exogenous variable.

There are other ways to compute elasticities. The ratio of percentage changes is the simplest, most basic approach.

## References

The economics literature on cigarette smoking is vast. Frank A. Sloan, V. Kerry Smith, and Donald H. Taylor, "Information, Addiction, and Bad 'Choices': Lessons from a Century of Cigarettes," *Economics Letters*, Vol. 77 (2002), pp. 147-155, is an accessible, informative starting point.

For a broader, historical review, see Allan M. Brandt, *The Cigarette Century: The Rise, Fall, and Deadly Persistence of the Product That Defined America* (2007).

# Appendix

1. The max run speed elasticity of distance on sand is 0.43 and this is quite inelastic, or unresponsive. Max run speed was doubled (so 100% increase) and distance on sand did increase, but only by 43%.

2. The IR elasticity of group size is $-1$ and this is unit elastic. IR fell by 60% (from 5 to 2) and group size rose by 60% (from 5 to 8). The minus sign means the two variables are inversely related.

3. The slope of $C^* = f(T)$ is 3/4, so multiplying this by 5/4 is 15/16 which does agree with the 0.9375 value in the text.

4. The slope of $C^* = f(T)$ stays constant at 0.75, but the elasticity increases from 0.9375 to roughly 0.9643. This shows that elasticity is a local phenomenon that changes depending on the value of the exogenous variable at which it is computed.

5. Governments hope that the cigarette demand is elastic, meaning that the price elasticity of demand is high. This way, small increases in taxes will produce big decreases in cigarette consumption.

6. Many studies have produced a variety of results, but the price elasticity of demand for cigarettes is expected to be inelastic, so less than one in absolute value. A good guess would be $-0.6$.

7. Teenage smokers are more price sensitive since they are not as addicted yet and typically have lower incomes than adults. If adults are at $-0.6$, teenagers might be at $-1.4$.