

Cost Minimization

You are on the factory floor of a manufacturing business. The CEO calls and says, “We need to make 300 units today.”

Your production function is simple: $Q = \sqrt{L} + \sqrt{K}$, where L is the amount of labor hours and K is the number of machines. You get to choose how much L and K to use, but you must produce 300 units.

You could, for example, use 90,000 hours of labor and no machines or 40,000 machines and 10,000 hours of labor. There are many other combinations of L and K that would meet the required quantity of output (q).

You have to pay for the inputs. The wage rate (w) is \$20 per hour and the rental rate of machines (r) is \$40 per machine.

Your goal is to find the L and K that minimize TC (total costs of production). Formally, your constrained optimization problem looks like this:

$$\begin{aligned} \min_{L,K} TC &= 20L + 40K \\ \text{s.t. } \sqrt{L} + \sqrt{K} &= 300 \end{aligned}$$

Implementing the Problem in Excel

You know that optimization problems always have three parts:

1. Goal (objective function) $\rightarrow \min TC$.
2. Endogenous variables $\rightarrow L, K$.
3. Exogenous variables $\rightarrow w, r, q$, and the function, $\sqrt{L} + \sqrt{K} = q$.

We will organize our spreadsheet with these parts, but first we want to visualize the problem.

STEP In cell A1 enter the label L and in cell B1 enter the label K . In cells A2 to A20 enter numbers 0 to 90000 by 5000. In cell B2 enter the formula $= (300 - \text{SQRT}(A2))^2$ and fill it down. Finally, make a *Scatter* chart of the data in cells A1:B20.

Your chart is displaying an *isoquant*. Since the prefix *iso* means equal (like an isosceles triangle), an isoquant shows all of the combinations of L and K that make the same amount of output.

There are many isoquants, one for each level of output. Your chart shows the isoquant for 300 units of output. It is the constraint for this optimization problem. The solution is on the isoquant, but we do not know which combination of L and K is the least expensive.

STEP Add a point on the chart and a scrollbar on the sheet to visualize that you can choose any combination on the isoquant. See the appendix if you need help.

So how do we find the cheapest combination of L and K that produces 300 units of output? With Solver, of course, but first we have to implement the problem in the spreadsheet.

STEP Label cells N1 and N2 as L and K , respectively. Cells N4, N5, and N6 have labels w , r , and q . Cell N8 is TC and N10 is the constraint. In cells M4, M5, and M6 enter the exogenous variable values. In cell M8 and M10 enter formulas. See the appendix if you need help.

Now you are ready to run Solver.

STEP Properly configure Solver and run it to find the optimal solution. See the appendix if you need help.

Solver should find a solution close to the exact answer of $L^* = 40,000$ and $K^* = 10,000$ with a $TC^* = \$1,200,000$. This solution makes sense since L and K are equally productive, but capital is twice as expensive as labor.

We can see that this solution is correct by computing the total cost of the points on the isoquant in columns A and B.

STEP In cell C1 enter the label TC. In cell C2 enter the formula $=20*A2+40*B2$ and fill it down.

It is easy to see that the cheapest way to make 300 units of output is the 40,000 and 10,000 combination.

Visualizing the Optimal Solution

Recall that the cigars and brandies utility maximization problems was visualized with a tangency condition between the utility contours and the constraint. Does the same apply here? Yes, it does.

We first create 3D surface and 2D contour plots of the total cost function, then we can draw a graph of the optimal solution.

STEP Copy cells A2:A20, insert a new sheet in your workbook, select cell A2 and paste. Select cell B1 and paste transpose (click the *Paste* down arrow and select the *Transpose* icon as shown in Figure 6.13). Enter a formula that computes the total cost for the given labor in column A and capital in column B. See the appendix if you need help. Make *3D Surface* and 2D contour plots.

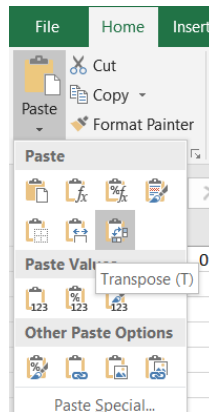


Figure 6.13: Paste transpose.

The surface is like a sheet of paper and the contours are straight lines. Each contour is called an *isocost* because it represents combinations of L and K with the same cost.

STEP Use Excel's *Shapes* group of drawing objects to add straight lines to your isoquant chart. Your lines must have a slope of 0.5 because w/r is 20/40. The lowest feasible isocost is a line with K -intercept of 30,000 and L -intercept of 60,000. All of your isocost lines must be parallel.

Figure 6.14 shows a visualization of the optimal solution. The curve is the constraint and the lines are isocosts. You want to be on the lowest isocost on the constraint.

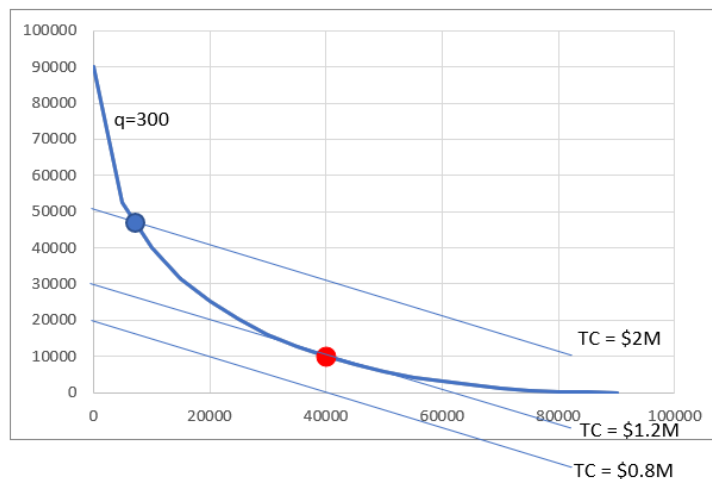


Figure 6.14: Visualizing the optimal solution.

There are many more isocost lines than the three shown. The point on the isoquant and the highest isocost is feasible in that you could make 300 units with this combination of L and K , but you would not be minimizing costs. The lowest isocost has lower costs than optimal solution (40,000 hours of labor and 10,000 machines), but since it is below the isoquant, it violates the constraint and no combination of inputs on that isocost is an eligible solution.

STEP Return to your chart and make sure you have an isocost line that is tangent to the isoquant at the optimal solution. Add a red dot (use Excel's *Shapes* objects again) at point $L = 40,000$ and $K = 10,000$ (as shown in Figure 6.14). Finally, improve Figure 6.14. What is it missing? See the appendix if needed.

Comparative Statics

We finish this problem by considering what would happen if the CEO called and changed the needed output level. We will compute the output elasticity of total cost.

STEP Use the Comparative Statics Wizard to explore how the optimal solution changes as you vary q from 300 to 400 by 10. Make a chart of the minimum total cost as a function of quantity.

You have made a graph of the cost function. This tells us how costs of production vary as output changes.

STEP Use your CSWiz results to compute the slope of the cost function and the output elasticity of total cost as q rises by 10.

The slope is rising slowly which agrees with the shape of your cost function (it is increasing at an increasing rate). The elasticity is also increasing, but very slightly. Of more importance is its value of a little over 2—total cost is quite responsive to output in this example.

Takeaways

Input cost minimization is a constrained optimization problem where we are asked to find the cheapest way to produce a given output.

Solver can do this problem and the Comparative Statics Wizard can be used to explore how the optimal solution changes as quantity changes.

The optimal solution is visualized as a tangency between the isoquant and isocost lines.

Notice how many other concepts and skills were repeated as you worked through this problem. Once you get the pattern, it is easier to understand other applications.

Appendix

To add controls to a spreadsheet, the *Developer* tab needs to be visible on the *Ribbon*. If it is not, click *File*, then click *Options*, then click *Customize Ribbon*, and check the *Developer* item.

STEP Copy cell range A2:B2 and paste it below your chart. To add this one point to your chart, copying, pasting, and editing the SERIES formula is an easy way to do this. You need to replace the x - and y -axis arguments in the pasted SERIES formula with the cell that has the x -axis value and the cell that has the y -axis value.

Next, we add the scrollbar control, but it has a maximum value of 30,000 so, as a simple workaround, we can connect the scrollbar to a different cell, say the cell below the 0 value and then change the 0 value cell to 10 times the cell below it. The scrollbar maximum should be set to 9000.

STEP Click the *Developer* tab on the *Ribbon*, click the down arrow in the *Insert* button, and select the scrollbar icon (in the top, form control group). Click and drag on the spreadsheet (roughly under the chart) to create the scrollbar control and link it the appropriate cell on your spreadsheet.

To use Solver to find the cost minimizing combination of L and K , cells M4, M5, and M6 are 20, 40, and 300. Cell M8 has formula $=M4*M1+M5*M2$ and should be formatted as \$. The constraint is in cell M10 with the formula $=SQRT(M1)+SQRT(M2)-M6$.

For Solver, the objective function is M8, the Min radio button should be checked, and the changing cells are M1 and M2. The constraint should be that $M10 = 0$.

The formula in cell B2 needed to make a 3D chart is $=20*A2+40*B2$. Format as \$ with no decimals and fill down and right.

Figure 6.14 can be improved by adding an appropriate title and labeling the axes.