Honor Scholar Thesis

EXTRACTION POLITICS:
AN ECONOMIC ANALYSIS OF RARE EARTH ELEMENTS

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Section 1: Backgrounder

Summary

Rare earth elements (REEs) have critical applications in high-technology industries such as defense and clean energy. China holds more than 90% of the current production capacity for these elements and in September, 2010 announced plans to cut its exports of the elements by 72%. Amid a rush to develop substitutes for the materials, in 2012 the U.S., E.U., and Japan have filed a case against China in the WTO.

This paper will analyze the extraction of rare earth element (REE) extraction from the joint perspective of Economics and Political Science. After a brief backgrounder provided in Section 1, Section 2 of this paper discusses the Economics of the issue. The topic is interesting from the perspective of Economics because REEs are a scarce and depletable resource. As a result, it is important to identify how quickly they should be extracted given economic factors such as costs, prices and the interest rate. Theories of optimal extraction from the Economic literature will be used to tackle these questions and using Microsoft Excel, China's economically optimal extraction path is charted, given such factors as mineral stocks, cost of extraction, price and interest rate

Section 2 assumes that profit maximization is the objective of the Chinese government but Political Science would tell us that this may be only part of a nation's objectives. Power, cooperative international relations or environmental sustainability might be other objectives that temper the results observed through a purely economic worldview. Section 3 therefore considers the politics of REEs from the perspective of the relationship between the China and the WTO and China's environmental movement with its government.

REEs have important strategic value in high-technology industries such as defense and green energy and while the Chinese government claims environmental protection as its reasons for export restrictions, skeptics suggest that China is violating international trade law for its narrow national interest. Given China's formidable growth and growing importance in international forums like the WTO, its violation of international law could weaken systems of international cooperation. Yet, China is a developing nation and

bound by the constraints and priorities that are unique to developing nations such as poverty alleviation, environmental sustainability and effective governance. In that context Section 3 discusses the possible outcomes of the REE case against China currently underway in the WTO and the consequences that ruling would have for international trade. It also touches on the role of the Chinese civil society, particularly its Environmental NGOs (ENGOs) in China's foreign policy and growth priorities.

Section I: Backgrounder

Introduction to Rare Earth Elements

Rare Earth Elements (REEs) are a group of 17 elements in the periodic table. 15 fall within the chemical group called lanthanides. These are lanthanum, cerium, praseodymium, neodymium, promethium, samarium, europium, gadolinium, terbium, dysprosium, holmium, erbium, thulium, ytterbium, and lutetium (Humphries, 2011). Yttrium and scandium are also classified as REEs but are not lanthanides (Humphries, 2011).

REEs can be identified as heavy or light, based on their atomic masses. The heavy elements, yttrium plus the elements from gadolinium to lutetium are scarcer but considered more "desirable" commodities by the USGS for their end uses (Humphries, 2011). In fact, REEs are important enough to be classified as critical materials by the U.S.Department of Defense. These are, "material essential for military equipment, unique in the function it performs, and for which there are no viable alternatives" (Grasso, 2011, p. 10). In the short-term, the elements dysprosium, neodymium, terbium, europium, yttrium and indium are considered most critical, both for their applications in defense and clean energy industries as well as the risk they face for supply disruptions (US DOE, 2010).

Rare earths elements have unique properties of magnetism, luminescence and strength that make them indispensable to the manufacture of high-tech and clean energy products (Tse, 2011). The following section discusses the end uses of REEs more thoroughly.

Uses

Rare earth elements are used in manufacturing, defense and science and technology sectors (Grasso 2011). 59% of global consumption of REEs comes from their use as catalysts, in glassmaking, lighting, and metallurgy (USGS 2011). REEs are also used as fluid cracking catalysts in petroleum refining, the manufacture of phosphors, automotive catalytic converters, flat panel displays, permanent magnets and rechargeable batteries (Humphries 2011).

41% of global consumption comes from the manufacture of battery alloys, ceramics and permanent magnets (USGS 2011). In relatively more mature markets lanthanum and cerium (both light rare earths) comprise 80% of REE demand whereas in emerging markets, 85% of all REE use is accounted for by heavy rare earths, dysprosium, neodymium, and praseodymium (USGS 2011).

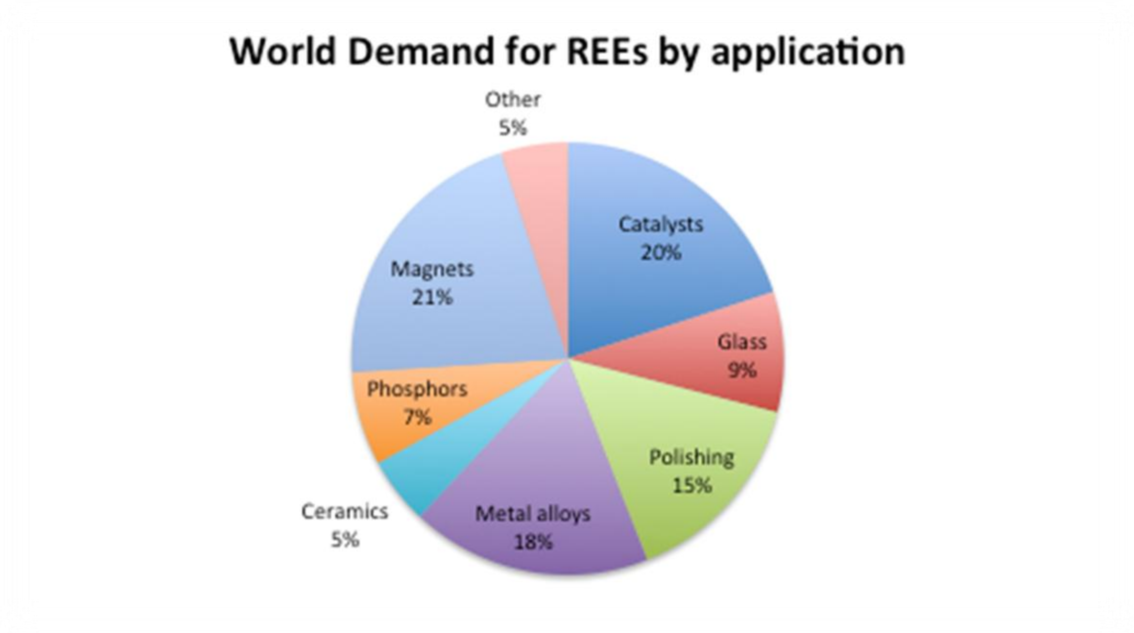

Figure 1: *World demand for REEs by application. Derived from Humphries (2011).*

Fluid cracking is the process of converting a heavy hydrocarbon into lighter hydrocarbons in the petroleum industry (USGS, 2011). In the glass industry, REEs are added to glass to change their absorption of ultraviolet light or refractive index, colorizing or decolorizing (USGS, 2011). Metallurgical applications of REEs include the addition of small quantities to aluminum, iron and steel to improve specific physical properties of alloys (USGS, 2011).

REEs like Europium and Terbium are also used to produce phosphors that are used in the production of cathode tube displays that are components of color televisions and fluorescent lamps (APS, 2011). In general their utility comes from their ability to convert incident radiation to a specific color (USGS, 2011). The REE Yttrium is used in energy efficient, solid-state lighting (APS, 2011). Photovoltaic technology like that described above is also used to produce solar energy (US DOE, 2011)

Among the more critical and specialized applications of REEs are in the manufacture of permanent magnets called Neodymium-Iron Boron or Samarium-Cobalt magnets (Grasso, 2011). Named for the heavy REEs neodymium and samarium that they contain, these permanent magnets create a stable magnetic field without an external power source (US DOE, 2010), making them a critical component of systems that convert electromagnetic energy to mechanical energy (APS, 2011). More specifically, they form an indispensable, light-weight addition to motors and generators of various sizes. Smaller motors, containing small quantities of REEs are used in everyday electronics like the disc drives of computers or car windows. The motors in electric and hybrid cars use about 200 gms of neodymium and 30 gms of dysprosium. Among the biggest users of REEs are wind turbine generators that use approximately a ton of neodymium in the permanent magnets in their motors (USGS, 2011).

While this information is classified, it is estimated that the Department of Defense is responsible for approximately 10% of the U.S. consumption of rare earths. (US DOE, 2011)

Military applications:

a) Guidance and control systems. Neodymium, Praseodymium, Samarium, Dysprosium and Terbium are used to produce compact permanent magnets that are used in the guidance and control systems of Tomahawk cruise missiles, smart bombs and predator unmanned aircraft (Humphries, 2011).

b) Targeting and weaponry. Yttrium, Europium, Terbium have applications in amplification of energy and resolution. They are used in lasers for enemy mine detection, underwater mines and interrogation.

Other rare earth elements are used in optical equipment, speakers, satellite communications, radar and sonar on submarines (Grasso, 2011).

Global Reserves and Extraction

Rare earths are more abundant in the earth's crust than copper, lead, gold or platinum however, they are in short supply because in most deposits, they are not of concentrations that make extraction economically viable (Humphries, 2011). This unavailability of REE minerals is compounded by the concentration of productive capacity in China, whose national interests diverge from the rest of the world, resulting in a global shortfall of supply. This section will focus on the geological and chemical circumstances that influence the availability and extraction of rare earth elements.

By one measure of geological scarcity, parts per million (ppm), the concentration of REEs present in the crust varies widely. Cerium is the most abundant of REEs, at 70 ppm. Lanthanum, Yttrium, Neodymium and Scandium are ranked 2nd to 5th in order of decreasing abundance. Their respective, estimated crustal concentrations are 39, 33, 41.5, and 22 ppm (USGS, 2010).

China holds the largest section of world reserves of REEs at 36% (approximately 43 million metric tons). The United States holds 13% (Humphries, 2011). South Africa, Canada, Vietnam, Greenland[1], Australia, Brazil, India, Russia, Malaysia and Malawi hold smaller portions of world reserves (Humphries, 2011; USGS, 2011).

REEs occur in alkaline igneous rock that is formed from the cooling and hardening of magma. In this case, elements may form simpler mineral structures when they fail to join with more complex mineral structures. In the process, economically viable mineral

[1] Greenland holds about 4 million metric tons of REO reserves in a very large but low-grade deposit of REEs. As yet, there are no plans to produce here which is perhaps why it is not discussed as often as Australia or Malaysia that are close to production.

deposits are formed (USGS, 2010). The specific REE mineral that is formed by this process is Bastnaesite (USGS, 2010).

REEs also occur in ‘placers’ that are formed from the erosion and deposition of metals into streams, rivers, deltas, and alluvial fans (USGS, 2010). The original source of placer deposits may be igneous (formed from cooling magma), sedimentary (formed from previous erosion, deposition and hardening) or metamorphic (formed from the chemical process of pressure or temperature) (USGS, 2010). Placers contain minerals like Monazite and Xenotime, which contain REEs that are deposited with other heavy metals.

The shared geological and metallurgical properties that make REEs a critical material are as follows:

A) Mineralogical and chemical complexity of ores

Most metals occur in minerals of only one phase. A phase can be understood as a chemical compound. For example, Zinc (Zn) deposits typically occur in the phase of Zinc Sulphide (ZnS) in the mineral Sphalerite (USGS 2010). In such cases, the metal can be extracted by one or a few simple process such as smelting (USGS, 2010). REEs on the other hand occur in multiple phases and each individual mineral deposit may contain a different combination of these phases. For example, the mineral Bastnaesite may occur as {(Ce, La, Y) CO_2F] or [(Ce, La)CO_3(OH, F)]. As a result, the extraction processes and technology may differ across mines. Opening a new mine requires the investment of time and money in testing new extraction techniques, thus making the process of opening a new REE mine a logistical and technological challenge much more expensive than opening a Zinc mine (USGS, 2010).

B) Radioactive byproducts

REEs often occur together with thorium and uranium which remain in the tailings of REE mines, causing these to be radioactive at “unacceptable levels” (APS, 2011, p. 12). The thorium and uranium are left in the tailings because they cannot be recovered at commercially viable costs. The problem of radiation is particularly acute in the mining of the minerals monazite and xenotime sands. In 1998, the REE mine at Mountain Pass, California was closed down due to a contamination from radioactive spills (APS, 2011). The costs of safely mining and handling radioactive minerals and the strict regulation of this in most of the developed world has contributed to higher costs of extraction and supply shortages of rare earth elements (USGS, 2010).

C) Produced as coproducts or byproducts of other extractive processes

REEs are produced as coproducts or byproducts of other extraction processes. A coproduct is a primary product of the refining process when there is more than one metal extracted. A byproduct is one which occurs incidentally to the production of the primary product. Here the costs of extracting and processing the byproduct are covered by the primary product (USGS, 2010). Because, REEs are not often the primary product of a mine or processing facility, their supply fluctuates with the supply of the primary product (USGS, 2010).

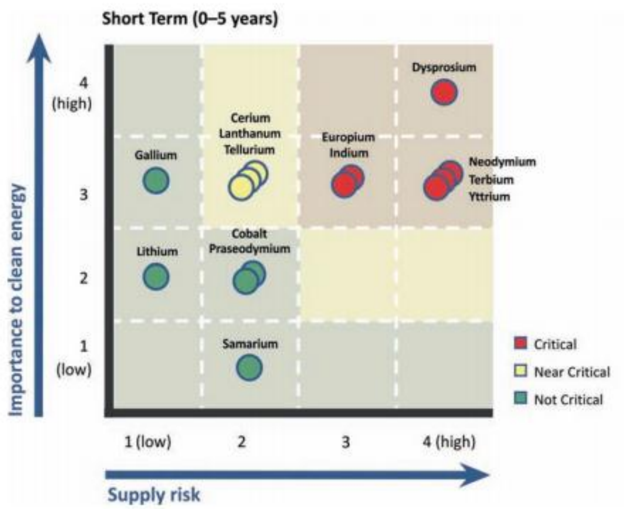

Figure 2: *A matrix describing the criticality of various key REEs over 0-5 years.(Image from US DOE 2011).*

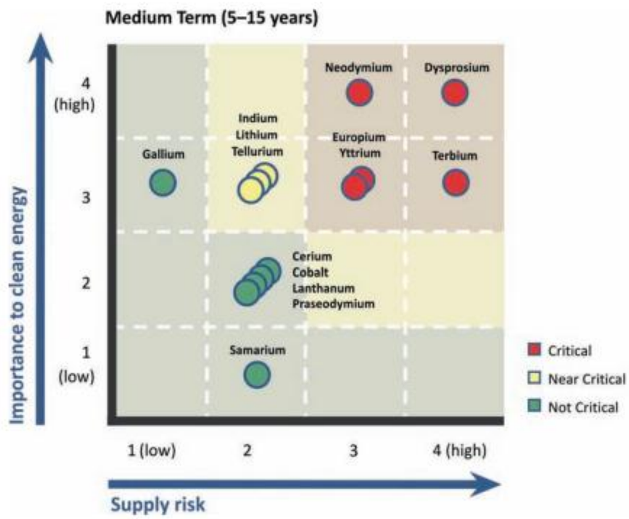

Figure 3: *A matrix describing the criticality of various key REEs over 5-15 years.(Image from US DOE 2011).*

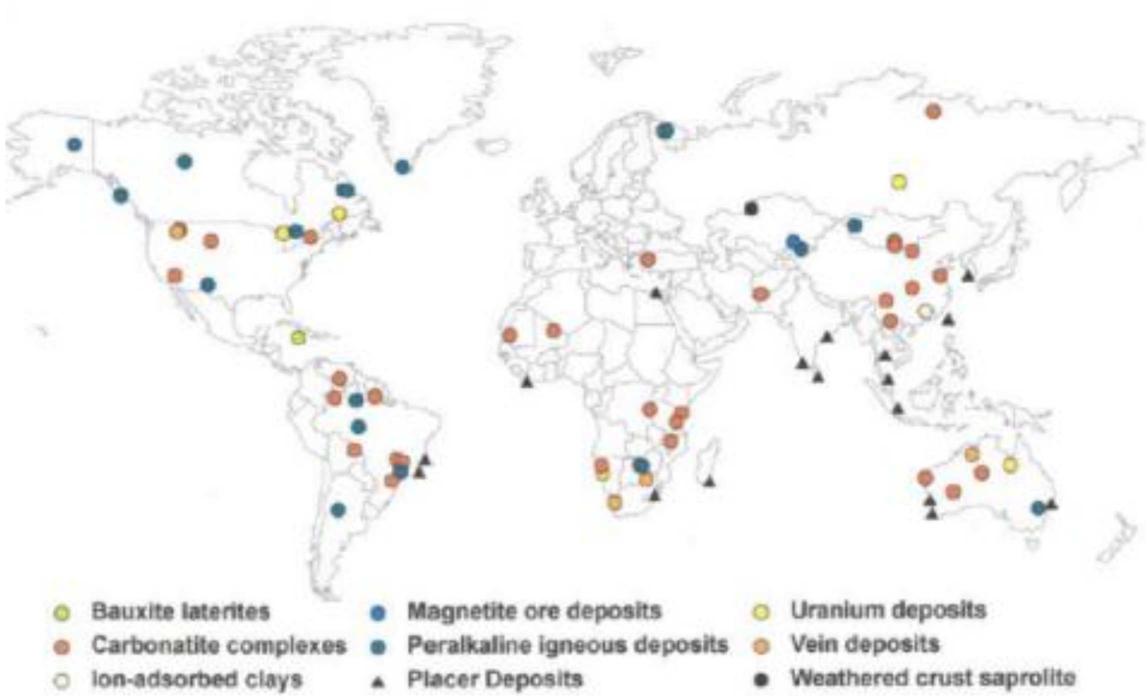

Figure 4: *The known worldwide distribution of REE ores (Image from APS (2011)).*

REEs experience four stages of production, which are as follows:

- Mining - Ore is removed from mineral deposits in the ground.
- Separation- Ore is separated into individual rare earth oxides.
- Refining - RE oxides are refined into metals of varying grades of purity.
- Forming - Metals are converted to rare earth alloys.
- Manufacturing - Alloys are manufactured into devices and components (Grasso, 2011).

Grasso (2011) estimates that rebuilding the REE supply chain in the United States, and other countries where rare earths have previously been mined could take up to 15 years and hinges on the capital investments in infrastructure, the development of new extractive technologies and processes, and the acquisition of intellectually property rights (such as patents) that are no longer held by these countries.

There is no shortage of REE deposits, but these deposits may never be used to produce REEs because of the complications of the metallurgical extraction process, absence of necessary infrastructure or environmental impact, in addition to the usual production limitations raised by labor costs, social or political instability (APS, 2011). China has become dominant in the REE market from its geological fortune at having exploitable reserves of REEs, the technical knowledge to extract material and low labor costs and environmental standards (APS, 2011). REE are located in deposits of minerals bastnaesite, monazite and xenotime (Humphries, 2011).

Rare Earth Export Restrictions and Dispute

Rare earth elements first hit the news in July 2010 when the Chinese Ministry of Commerce announced that it would begin to cut its exports of the material by 72% (Grasso, 2011). In September of the same year, China cut exports of rare earth elements to Japan, over a maritime dispute (Nayantara, 2011). While the Chinese government alleged that this export cut was a result of the decisions of private firms, the action nonetheless sent shockwaves through the international community, as it drew into question the supply security of these critical materials.

Figure 5, below, shows the price of the rare earth element Neodymium over a three year period. From 2009 to late 2011, prices of REEs rose, some more than thirty times. As of late 2011 however, these prices have been falling in international markets as a result of reduced demand (Bradsher, 2011).

Figure 5: Price of the rare earth element Neodymium in international markets. Prices increased drastically from 2009 to 2011, and then declined (Bradsher 2011).

The Chinese government justifies its use of rare earth quotas as necessary to protect its environment and encourage the growth of a sustainable domestic processing and manufacturing industry for rare earths (APS, 2011). Outside China, the opinion is that REE export barriers are an attempt by the Chinese government to establish control over a critical material and control prices (APS, 2011).

These export restrictions have come in the form of export quotas, taxes, production quotas, the withdrawal of VAT refunds on exports and a ban on foreign investment in rare earth resources and mining (Nayantara, 2011). These export restrictions are problematic because measured in rare earth oxide content, China produces 97% of the world's rare earth elements. China has cut exports of the material from 50,000 metric tons in 2009 to 30,000 metric tons in 2010 (Humphries, 2011).

Global production of REEs in the form of oxide is approximately124,000 tons. The difference between that quantity and demand is met by the processing of previously mined stocks. By 2012, world demand is projected to rise to 180,000 tons annually (Humphries, 2011). However, given the metallurgical complications and significant investments necessary to begin new rare earth mines, these are unlikely to begin production for another 10 or 15 years (Humphries, 2011). The USGS estimates that in the long run global reserves and estimated undiscovered resources will be sufficient to meet demand for rare earths. However, in the short run, the shortage of REEs is negatively affecting manufacturing facilities around the world (Humphries, 2011).

The odd part about the insecurity over rare earth elements in the past two years is that Chinese near-monopoly of this industry has not occurred overnight. China's R&D facilities and mining monopoly have come about through "systematic government policy" over the past 50 years (Humphries, 2011, p. 9). Tse (2011) notes that that through the 1990s and 2000s, the Chinese government and its rare earth producers have met to discuss ways to control production and restrict exports to secure domestic supply and conserve the environment. China's production grew 40% from 1978 to 1989 and between 1996 and 2006 output grew from 2,600 to 39,000 tons (Humphries, 2011). In 1990 China contributed 27% of world output of REEs, but by 2008 this had risen to more than 90% of world output (Tse, 2011). In 1990 its exports of REEs grew and lower prices outcompeted mines and processing facilities in other countries like the U.S. and Australia (Humphries, 2011).

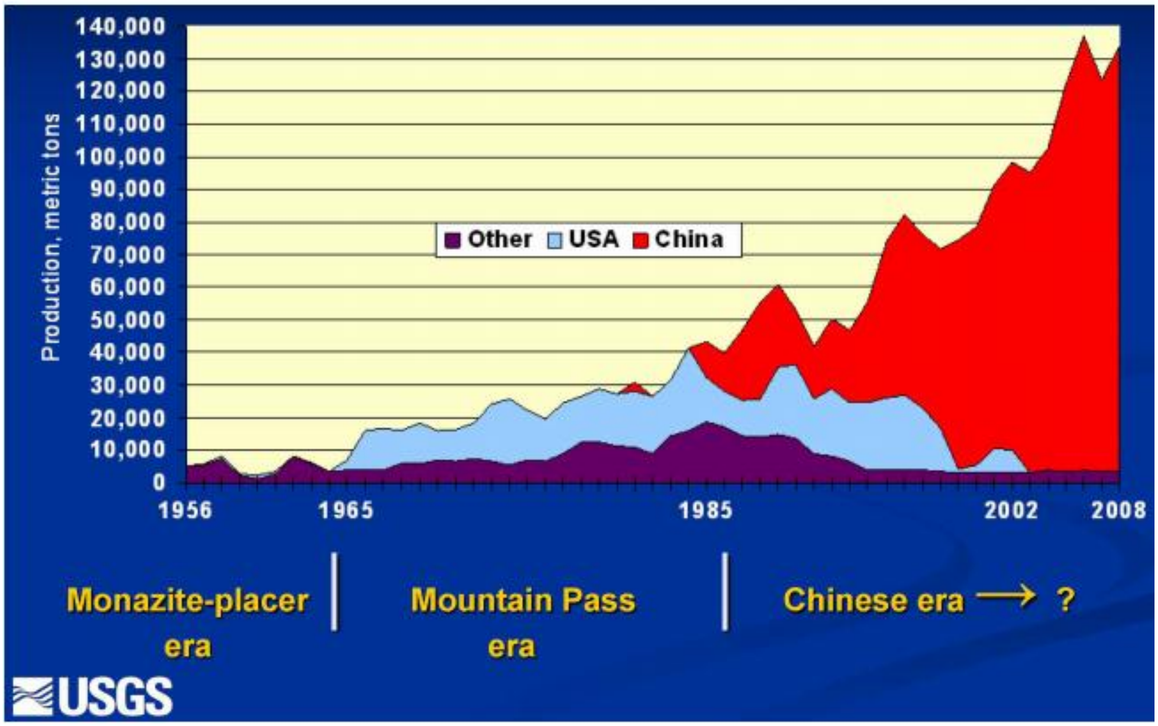

Figure 6: *Share of production of rare earth oxides (REOs) from 1956-2008. (Tse, 2011).*

Concurrently domestic demand for the element grew 380% from approximately 19,000 tons in 2000 to 73,000 tons in 2009 (Tse ,2011). China's growing internal demand led it to secure its production for domestic manufacturers of high-technology goods like consumer electronics and wind turbines (Humphries, 2011).

Chinese government policy also began to change in the early 1990s, to secure the domestic supply of rare earths. In 1990, REEs were declared a protected and strategic material in China and foreign investors were prohibited from mining it (Tse, 2011). Plans were created for overall production quotas, as well as province and mine-specific quotas (Tse, 2011). Production in excess of the quota restrictions was conducted by unlicensed mines with suboptimal technology leading to environmental damage and waste (Tse, 2011). Part of the Chinese government's current defense of export restrictions comes from its crackdown on illegal, polluting mining.

Chinese policy has not concentrated solely on mining and processing of REEs, but also considered the downstream, manufacturing industry. The government has systematically discouraged the export of raw materials in favor of manufactured goods (Tse, 2011). The most drastic policy in this regard before the export quotas implemented in 2010 was the elimination of rebates on rare earth exports in 2005 and the ban on trade of many rare earth commodities (Tse, 2011).

The United States was the largest producer of rare earths in the period 1960-1990 after which production shifted to China as a result of lower labor costs and environmental standards (Grasso, 2011). However, relying on Chinese imports of rare earths was economically sound at the time because China produced at lower cost and had a competitive advantage in REE production. The oversight was that over time sources of REE supply gradually became less diverse “in number and location”, giving China monopoly power over REEs (APS, 2011).

After China abruptly stopped imports of rare earths in 2010, Japan has also been conscious of its reliance on China and begun to look for alternative suppliers and substitutes to the elements. Japan is currently negotiating agreements with Australia, Mongolia, and Vietnam in preparation for rare earth shortages (Nayantara, 2011).

The U.S. government, like Japan and Korea, is now considering stockpiling REEs among other critical elements (APS, 2011). In 2012, the U.S. firm Molycorp will construct a manufacturing facility and open production at a mine that was closed down in 1998. This mine at Mountain Pass, California will produce approximately 40,000 tons of REEs oxides by 2013 but will lack the processing capacity to convert REOs to metals (Grasso, 2011).

In the next few years, the following mines outside China will begin to reopen (Nayantara, 2011):

- Molycorp mine in Mountain Pass, California U.S.A
- Mount Weld, Nolan’s Bore and Dubbo Zirconia, Australia
- Unspecified name, East Coast, Brazil
- Nechalacho and Hoidas Lake, Canada

- Dong Pao in Vietnam

Unfortunately, these mines hold high concentrations of lighter rare earths where heavier rare earths are rarer and more valuable (Nayantara, 2011).

Additionally, an Australian owned refinery in Malaysia that was scheduled to open in late 2011, is now scheduled to open in late 2012 (Bradsher, 2012). This project has been plagued by technical delays and opposition from Malaysian civil society over disposal of radioactive waste from the site. The refinery that has the capacity to meet about a fifth of world demand for rare earth elements (Bradsher, 2012).

In summary, REEs are critical and in short supply in the short-run, until new mines and refineries come into production. Meanwhile, downstream industries that use the elements, like the permanent magnet and automobile industries are struggling to find substitutes. The relatively obscure rare earths industry was pushed into the spotlight by China’s export restrictions on the minerals that have major economic and political dimensions. The following sections, 2 and 3, will discuss each of these dimensions in detail.

Section 2: An Economic Analysis of Rare Earth Elements

Section 2: An Economic Analysis of Rare Earth Elements

Literature Review

Hotelling (1931) provides a seminal contribution to the exhaustible resource literature by tackling the central question that the field aims to answer, namely, what the optimal rate of extraction of a resource should be given that it is finite and time is not. Hotelling (1931) says, "it may seem that the exploitation of an exhaustible natural resource can never be too slow for the public good" and yet goes on to prove that in the case of a perfectly competitive market, the time period of optimal extraction is finite and under monopoly, the optimal extraction of the resource could be finite or infinite depending on the demand for the resource (p. 138).

Hotelling (1931) identifies two objectives in the extraction of resources, that of the private firm in a perfectly competitive setting or monopoly, and the social objective of extraction. In the first case the owner of the resource is assumed to aim to maximize the present value of profit from resource extraction. In the second case, social utility is a function of the extraction in each time period, price and the total time in which the resource is extracted. In this case Hotelling (1931) finds that monopoly production causes production to be slower and prices to be higher, which does not maximize the social value of the resource.

The industrial structure of extraction is defined by the number of firms in the market and their reactions and responses to price. In the case of perfect competition, price is determined in the market and is constant to the individual producer. However, under the monopoly condition, the producer has some control over prices although price is also determined by demand for the resource. Hotelling (1931) models price under perfect competition as constant in each time period, but discounted to the present. Therefore, in each successive year of extraction, price is discounted by a function of the interest rate such that the present value of profit in the future is negligible. This ensures that the optimal time period of extraction is not infinite but finite and can be predicted based on prices, interest rates and the quantity of the resource available.

Hotelling's finding that prices must rise at the rate of interest rate is known in the subsequently as the 'Hotelling r-percent rule' (Toman, 1986). Presented as $p_{t+1}=(1+r)p_t$ where p_t is the present time, p_{t+1} the subsequent time period and r the rate of interest, this rule is applied in discounting future revenues and profits to the present (Coats, Pecquet & Sanders, 2009). An alternative and intuitive understanding of this condition for optimality is that discounted prices must remain constant across time. Without this condition, a producer would move some extraction into time periods with a higher price after discount in order to increase the present value of profit. This adjustment between time periods would cause prices to adjust such that present value from each period would be equal (Coats et al., 2009).

In the case of monopoly extraction of an exhaustible resource, Hotelling (1931) finds that production is extended over a longer period of time because prices are sensitive to production, and the monopolist keeps prices artificially high in order to extract consumer surplus. Yet, even in this case the optimal time period of extraction is finite although extraction occurs over a much longer time period than under perfect competition. Hotelling (1931) theorizes that the optimal extraction path for a monopolist would be downward sloping, convex to the origin, indicating that extraction in future periods gradually reduce to zero. In that case prices would continue to rise until the last time period of extraction at which time the market would reach the 'choke price' or maximum price that consumers are willing to pay. The only case in which extraction would not eventually become zero is the case where prices continue to rise at the rate of interest all the way to infinity. In this theoretical case, extraction would continue into infinity and the extraction path would be asymptotic to the x axis. This is illustrated in the graph below.

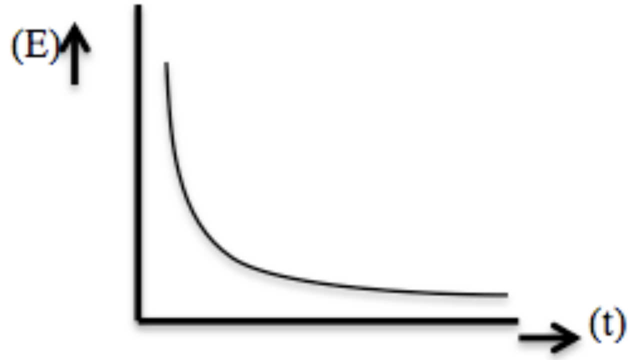

Figure 1: *This graph tracks a monopolist's optimal extraction path with extraction (E) on the y-axis and time (t) on the x axis. Hotelling (1931) finds that in some cases optimal extraction under monopoly could stretch across infinite time.*

What Hotelling (1931) therefore concludes is the amazing result that given a high enough price and a positive discount rate, an exhaustible resource may never be exhausted. It may become scarcer over time, but as long as the price that consumers are willing to pay continues to rise, extraction of the resource need never cease. This finding was revelatory at the time that Hotelling (1931) presented it and it has continued to serve as a starting point for further work in the economic modeling of exhaustible resources.

Hotelling's analysis begins with the case of perfect competition, continues to describe the case of monopoly, the case for maximizing social welfare and finally concludes with a discussion of an oligopoly market for exhaustible resources. A similar organization is used in Sweeney (1993) and will be used in the analysis of the case of rare earth elements in this paper.

While Hotelling's "r percent rule" is used as the basis for more complex models, it assumes resource homogeneity and extraction costs independent of remaining stocks, new discoveries and investment (Toman, 1986). These assumptions have major implications for modeling costs in the extractive industry. The assumption that extraction costs are independent of remaining stocks implies that the marginal cost of extraction is constant and could therefore lead to incorrect predictions of:

a) the time period in which a resource may be exhausted or

b) the distribution between current and future extraction.

Toman (1986) corrects this assumption by introducing increasing costs from increased rate of exploitation and a decrease in stocks. The theory is that stocks are non-homogenous and therefore the more easily extractable stocks are depleted first. Resources that are extracted in later periods are harder to access and therefore more costly to extract. Toman (1986) calls these increasing costs from extraction ‘depletion effects’. These depletion effects then become part of ‘user costs’ of the resource, which are like an opportunity cost of extracting the resource. When the resources are extracted, the costs of any future extraction go up, therefore, the cost of extraction in the present also contains an implicit opportunity cost that Toman refers to as ‘user costs’. In effect then, the depletion effect causes marginal costs in subsequent periods to rise.

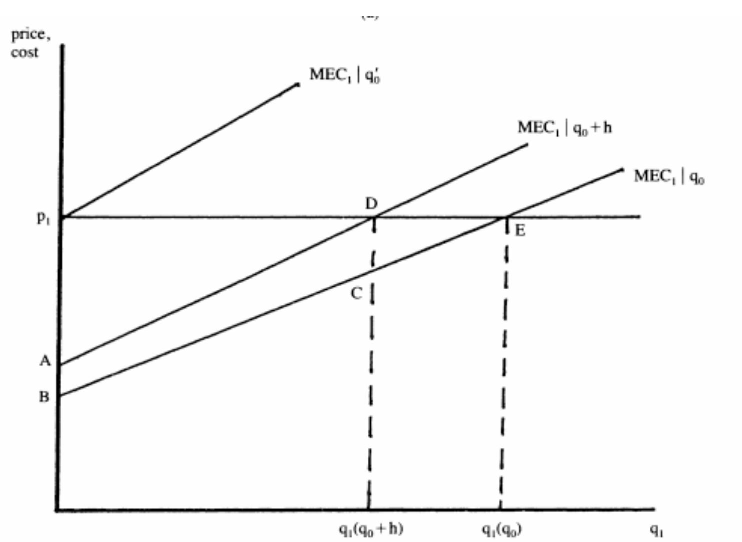

Figure 2: *Toman (1986). Marginal extraction costs (MEC) rise in each subsequent period by the user cost (U). Eventually user costs are high enough that extraction (q) falls to zero.*

Sweeney (1993) adds to this theory of costs, the possibility that costs could potentially fall as stocks decline if the exploitation of the resource itself furthers improvements in technology (Sweeney 1993). The model in Toman (1986) also assumes that increased rates of extraction drive costs up such that an increase in extraction in period one makes extraction in period two more difficult. The result is that with extraction in period one large enough, the cost of extraction in period two could be prohibitively high such that extraction does not take place (Toman 1986).

Sweeney (1993) builds on the analysis of Toman (1986) to suggest further costs related to depletion. Here costs are a function of the rate of extraction, a function of the remaining stock of the resource or some combination of those two components. The effect of remaining stock on costs is referred to as 'stock effects' and are theoretically justified because, as Toman (1986) suggests with 'depletion effects', resources that are extracted later may be of lower grade or less accessible, causing their marginal cost to rise (Sweeney, 1993). Sweeney also discusses opportunity costs, similar to user costs. Because profits from future extraction are discounted to the present, extraction rates become smaller over time. In this case opportunity costs come from forgoing future extraction in favor of the present (Sweeney, 1993).

Sweeney (1931) discusses three methods of solving the extraction problem. First, Lagrangian optimization second, its variant, the Kuhn-Tucker conditions and third, feasible variations. All of these are analytical and model the problem in a system of equations describing an equilibrium condition that is then solved to arrive at optimal solutions. This thesis will apply those approaches in Excel to chart extraction paths for rare earth elements. Additionally, it will use the numerical approach of Excel's solver to manually create optimal extraction paths that can be compared against the other approaches.

Conrad (1999) implements Hotelling's r-percent rule under perfectly competitive conditions to derive optimal extraction time paths under linear demand curves and demand curves with constant elasticity. An interesting finding in Conrad's exposition of the monopoly case is that Hotelling's r-percent rule holds with a demand curve of constant elasticity. Because the r-percent rule holds and the monopolist must extract such that the price rises with the discount rate, such a monopolist behaves identically to a perfectly competitive producer. Conrad (1999) like Sweeney (1931) finds that extraction under a under a perfectly competitive producer provides greater social welfare than extraction under a monopoly facing a linear demand curve.

Conrad (1999) differs from Hotelling (1931), Toman (1986) and Sweeney (1993) in discussing extraction across discrete time periods using the Lagrangian method rather modeling continuous time. Discrete time periods and a Lagrangian optimization problem will be used to solve the extraction problem in this paper.

Altogether, the theory of exhaustible resource extraction discussed here forms the theoretical basis for the functional forms and analysis in this thesis. The division of the economic analysis in this paper into a section analyzing the extraction dynamics in a perfectly competitive market, where price is exogenous, and then in a monopoly market where price is endogenous, also follow Sweeney (1993) in the discussion of those scenarios. Developing the case for perfect competition serves as a basis from which modifications may be made to represent the more realistic monopoly case for rare earth elements. This paper aims to add to the literature of exhaustible resources by applying theoretical models and graphical illustrations to understand the specific case of rare earth elements.

The Problem

Rare earth elements are classified as depletable resources because they are replenished so slowly that their stock is understood as fixed for the purposes of economic modeling and decision making. These elements are also expensive to extract, placing a further constraint on their supply (USGS 2010). In this scenario, the extraction problem consists of two parts. First, how much of the element should be extracted in each time period? Second, how long a time period should be considered?

The two questions are interrelated since the resource is finite. Therefore, the optimal extraction for each year in a five-year period would differ from the optimal extraction for each year in a hundred year period or even an infinite period. The optimal solution in determined by factors such as price, initial stock, cost of extraction and the rate of interest (or discount) that would apply to the stocks remaining in the ground.

Overview of Methods to Solve the Problem

Two approaches, analytical and numerical, can be used to solve this extraction problem. Analytical solutions discussed here are the Lagrangian, Kuhn-Tucker conditions which gives rise to a slightly modified Lagrangian, and Feasible Variations. Excel's Solver is the numerical method used to solve the REE problem.

1) Analytical Solutions – This is a broad category of solutions that model the problem mathematically to obtain an optimal solution using maximizing and minimizing calculus. Here the interaction between the variables of cost, revenue and the constraint are modeled mathematically and solved using a Lagrangian function to obtain a general solution for optimal values. If the symbols in the general solution are replaced with numerical values, then values for optimal extraction may be derived from this setup.

Lagrangian - Conceptually, the Lagrangian function can be understood to add a layer of complexity to optimization calculus by allowing the user to solve constrained optimization problems. This method incorporates the constraint into the objective function such that first-order methods of solving it can still be applied.

The class of analytical solutions is broad because mathematical models of varying degrees of complexity can be used to model extraction. For example, this analysis of the problem assigns cost a quadratic functional form. But a case could be made for assigning a different functional form to costs, which would present a slightly differently analytical solution.

Kuhn-Tucker Solution- Kuhn-Tucker solutions can be understood as a conceptual next step from the constrained analytical solution presented by the Lagrangian approach. Here multiple inequality constraints may apply such that one or more of the applied constraints becomes non-binding at a given time (Chiang 1984).

A constraint is said to be binding when changing the value of the constraint changes the optimal solution. When a constraint is non-binding or slack, then changing the value of the constraint does not change the optimal solution. In this case the constraint takes the form of an inequality and an interior solution is possible such that the constraint need not be fully exhausted (Chiang 1984). Here, we could obtain an optimal solution that satisfies the constraints but does not deplete the stock of material to zero.

This property is known as complementary slackness and takes the following form:

$$\lambda * (g(x) - c) = 0$$

Here, the relationship between the Lagrangian multiplier (λ) and the constraint $g(x)=c$ is such that one or the other is always equal to zero and therefore non-binding. If the constraint were fully exhausted then $g(x)-c=0$ and then $\lambda \geq 0$ must be true such that changing the constraint would change the optimal solution. This must be true because a negative value for λ would mean that reducing the value of the constraint produces an increase in the optimal solution which would mean that the previous optimal was not maximizing.

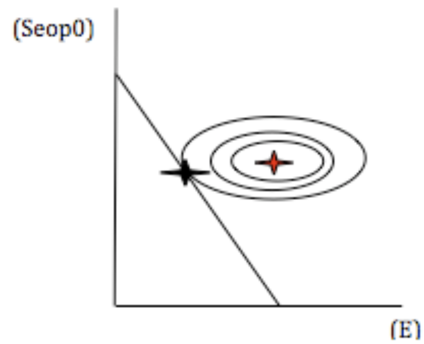

Figure 3 *:λ≥0. Constraint is binding. Derived from Osborne (2011)*

However if the constraint were not fully exhausted then λ would take on a value of zero to represent the unresponsiveness of the optimal solution to changes in the constraint.

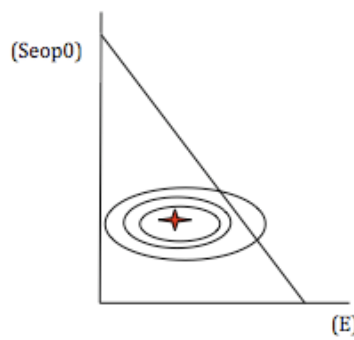

Figure 4: Constraint is slack or non-binding. The optimal solution lies within the constraint. Derived from Osborne (2011)

Once constraints are identified, the problem is converted to a Lagrangian function which is solved for optimal values of the variables and Lagrange multipliers. The value of the Lagrange multiplier is "equal to the rate of change in the maximal value of the objective function as the constraint is relaxed" (Osborne 2011). In other words, this multiplier measures how responsive the optimal solution is to changes in the constraint function.

This paper will implement an analytical solution to the extraction problem using a particular cost function with exogenous prices. The analytical solution implemented in excel, will allow a user to vary exogenous factors such as price, the interest rate, and the time period to determine what the optimal extraction path would be in each case. The final excel workbook will also contain sheets that are partially calibrated to the case of China and rare earth elements in terms of prices and annual extraction.

Feasible Variations –The method of feasible variations determines a set of conditions in which it is impossible to derive an improvement in the output function by implementing any feasible variations in the input functions.

In the case of an extraction problem, this would entail that a trajectory of extraction values across time would only be optimal if no variation in extraction in any time period, in the form of an increased extraction in one period and decreased extraction in another would produce an increase in profit.

The necessary condition for a trajectory to be optimal would be that the difference between present value of the price and marginal cost be equal in each time period. If this difference was larger in some period relative to another, then extraction would shift to the period with a higher difference and in doing so increase profits (Sweeney 1993). This would be a case of a feasible variation yielding higher returns than the extraction path earlier thought to be optimal. The difference discussed above could be understood to be a form of opportunity cost. Opportunity cost in its basic form is the cost of forgoing the returns from the next best alternative to a particular choice. In this case, forgoing extraction in one period in favor of another would cost the difference between the present value of price and marginal cost in the period forgone.

2) ***Numerical solutions*** **–** Numerical solutions use trial and error to find the optimal solution. In the case of the extraction problem, these methods substitute different values for extraction in each period until they reach the solution that satisfies the constraints and the objective of maximizing profits.

Excel's Solver – When provided with a set of exogenous variables that affect the independent variable, Excel's Solver plugs in its own values for the endogenous variables, oscillating between values that are larger and smaller than the optimal value and comparing the resulting output. Using this method, it converges on the optimal values of input variables to arrive at maximum or minimum values of the objective function.

Excel Implementation

The following subsections, from 2.0-2.14, will apply economic theory to develop models in Excel which facilitate a clearer understanding of the mechanics of optimization and the visualization of optimal extraction paths. They begin with a discussion of a perfectly competitive firm considering two periods. Next they consider a perfectly competitive firm considering multiple time periods. When the analytical processes are established, the monopoly model is introduced, first in two periods and then multiple periods. Finally, modifications are made to price, interest rates and initial stock to illustrate how they affect optimal extraction paths.

2.0 Perfect Competition in 2 Periods

The *PC 2 Period* sheet describes the simplest case of an extraction problem, in which the industry is assumed to be perfectly competitive (PC) and in which the firm considers extraction in only two periods. Here, the firm cannot change the stock of the resource and rate of interest which are therefore exogenous variables. Optimal extraction in each period is the endogenous variable and depends on the values of the exogenous variables. Changing the values of one of the two exogenous variables, and holding the other constant would modify the extraction problem and produce a new optimal extraction path. Observing the response of the endogenous variables to changes or “shocks” to the exogenous variables is known as comparative statics and will be discussed in later sheets of this workbook.

The optimal extraction problem can be solved numerically, using Excel’s Solver and analytically, using a Lagrangian. An earlier section described these methods in general conceptual terms. Here these methods are described in greater detail, and applied to the specific parameters and assumptions of a PC, 2 period setup.

Solver Method

- Cost takes the following form: $TC_i = c_{0i}E_i^2 + c_{1i}E_i$; where i=0,1
- Here c_0 and c_1 are constant parameters that define the cost function. E_i represents the extraction in each of the two time periods.
- Revenue is the product of price and extraction: $TR = P_iE_i$
- Present values of revenue and cost are obtained by dividing revenue and cost in each period by the factor: $(1+r)^t$
- Here r is the rate of interest and t is the time period under consideration.

The problem is set up as displayed in the Figure below:

	A	B	C	D	E	F	G	H	I	J	K	L	M
1													
2	Initial S	20	beginning stock of resource										
3	r	5%	interest rate										
4													
5						Total extractio		Sum Rev	Sum Cost	Sum Profit	Sum PV Profit		
6						20		140	60.003	79.997	78.0979		
7	Time	Price	c0	c1	Stock (BOP)	E	Stock (EOP)	Revenue	Cost	Profit	PV Profit		Analytical
8	0	7	0.25	0.5	20	10.073171	9.9268	70.5122	30.404	40.108	40.1084		10.0731707
9	1	7	0.25	0.5	9.9268	9.9268293	0	69.4878	29.599	39.889	37.9894		9.92682927
10													1.46341463

Figure 5: *Excel setup in which Solver is allowed to maximize profit subject to constraints.*

To access Excel's Solver we must first install the Add-in. To do this go to Excel's *Options>Add-ins,* select *Solver Add-in* and click 'Go'. In the pop-up window that appears, check the box by *Solver Add-in* and then click 'finish'. Solver should appear under the Data tab in Excel. Solver will be used consistently in subsequent sheets, so this is the best time to make sure it is added to Excel and in working order.

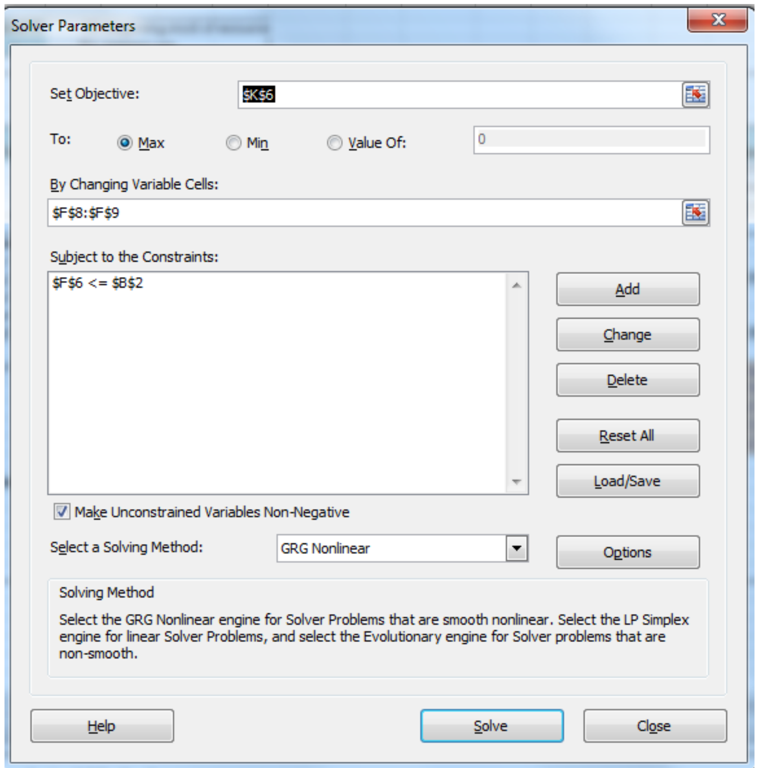

Figure 6: *Excel's Solver is set to maximize the sum of present value profit, in cell K6 by changing the values of extraction in cells F8:F9.*

The constraints imposed on Excel are: $(i)F6 \leq B2$. This says that total extraction, displayed in cell F6 must be less than or equal to the total stock of the resource available. $(ii)F8{:}\,F9 \geq 0$. This says that extraction in each period must be greater than or equal to zero. It will not allow Solver to choose negative values of extraction.

As shown in Figure 5, Solver's results are displayed in cells F8:F9. Solver tells us that the optimal extraction in time periods one and two are 10.07 and 9.92 units respectively.

Lagrangian Analytical Method

Where Excel's Solver arrives at the solution using trial and error and eventually converges on the correct solution, the Lagrangian analytical method uses optimization calculus to solve the problem. Here, the Lagrangian value L is maximized.

The Lagrangian for the two period optimization problem takes the following general form:

$$MaxL = \sum_{i=0}^{1} \frac{PE_i - CE_i + \lambda_i (S^0 - E_i - S^i_{EOP})}{(1+r)^i}$$

Where: P – Price per unit, C – Cost per unit, S^I – Initial stock, i – Time period in which extraction is taking place, E^i – Extraction in time period i.

Taking the first-order partial derivatives of the Lagrangian with respect to each variable gives the following 4 equations.

Above, equation (2) is the specific form of equation (3) for the first time period, i=0, so the denominator $(1+r)^0=0$. Equation (3) refers those situations i=1,2,3,4,…., t-1 where t is the number of time periods under consideration. For a problem with 100 time periods 'i' would take values from 1 through 99. Equation (4) has values for time periods from i=0 through i=t-2. This is because stock is assumed to be fully depleted by the final period, such that $S^t=0$. In a problem considering 100 time periods, the stock at the end of the hundredth period, S^{100} would be zero.

$$(1)\ \frac{\partial L}{\partial E_i} = \frac{P - C - \lambda_i}{(1+r)^i}$$

$$(2)\ \frac{\partial L}{\partial \lambda_0} = S^I - E_0 - S^0_{EOP}$$

$$(3)\ \frac{\partial L}{\partial \lambda_i} = \frac{S^i_{EOP} - E_i + S^{i-1}_{EOP}}{(1+r)^i}$$

$$(4)\ \frac{\partial L}{\partial S^i_{EOP}} = \frac{-\lambda_i}{(1+r)^i} + \frac{\lambda(1+i)}{(1+r)^{1+i}}$$

With the constraints specified in this problem (refer to the Excel screenshot above), the Lagrangian takes the following specific form:

$$MaxL = 7E_0 - \frac{1}{4}E_0^2 + \lambda_0(20 - E_0 - S_{EOP}) + 7E_1 - \frac{1}{4}E_1^2 + \lambda_1(S_{EOP} - E_1)$$

Note that the interest rate is assumed to zero in this example, so profits are not discounted to the present. Also, in the second part of the expression we see that the stock of the resource is fully exhausted, therefore the end of term period for the second period (i=1) does not exist. S_{EOP} from the first time period (i=0) is the initial stock for the second period.

Taking the first-order partial derivatives of the Lagrangian with respect to each variable gives the following 5 equations.

$$(1)\frac{\partial L}{\partial E_0} = 7 - \tfrac{1}{2}E_0 - \lambda_0$$

$$(2)\frac{\partial L}{\partial S_{EOP}^0} = -\lambda_0 + \lambda_1$$

$$(3)\frac{\partial L}{\partial \lambda_0} = 20 - E_0 - S_{EOP}^0$$

$$(4)\frac{\partial L}{\partial E_0} = 7 - \tfrac{1}{2}E_1 - \lambda_1$$

$$(5)\frac{\partial L}{\partial \lambda_1} = S_{EOP}^0 - E_1$$

Each derivative is set equal to zero as in standard optimization calculus and the system is then solved for values of the variables that would maximize the value of the Lagrangian. The resulting optimal values of all variables are denoted by asterisks.

In the case above equations (2) and (5) are substituted into equations (3) and (4) to give:

$$7 - \tfrac{1}{2}E_0 = 7 - \tfrac{1}{2}(20 - E_0)$$

$$\Rightarrow E_0^* = 10$$

Other optimal values are obtained by substitution, with the following results:

$$S_{EOP}^{0*} = 20 - E_0^* = 10$$

$$E_1^* = S_{EOP}^{0*} = 10$$

$$\lambda_1^* = \lambda_0^* = 7 - \tfrac{1}{2}E_0^* = 2$$

Above the system of equations was solved using regular algebraic methods. However, matrix multiplication provides a less cumbersome method of solving the system. It can be implemented in Excel and makes solving the problem much easier when the number of time periods increases and many more equations describe the system.

The principle of matrix multiplication holds that if Ax=b where A, x and b are matrices, then then $x=A^{-1}b$, where A^{-1} is the inverse of matrix A. In Figure 7 below, matrix A describes the system of equations (1) through (5). Each row represents an equation while each column represents a variable. The matrix therefore contains the coefficients of each variable in each equation. For example, the cell corresponding to column E_0, row eq1 contains 0.5, which is the value of the coefficient of E_0 in equation (1).

Matrix A	Eo	E1	Lambda0	Lambda 1	Seop0
eq 1	0.5	0	1	0	0
eq2	0	0.5	0	1	0
eq3	0	0	-1	1	0
eq4	1	0	0	0	1
eq5	0	-1	0	0	1
A^{-1}	1	-1	1	0.5	-0.5
	-1	1	-1	0.5	-0.5
	0.5	0.5	-0.5	-0.25	0.25
	0.5	0.5	0.5	-0.25	0.25
	-1	1	-1	0.5	0.5
Matrix B	7		Matrix x*	E0*	10
	7			E1*	10
	0			Lambda0*	2
	20			Lambda1*	2
	0			Seop0*	10

Figure 7: *Matrices A, B and x implementing the analytical solution for 2 periods. Discount factor disregarded.*

Vector b contains the values of the constant term from each equation. Vector x* contains the values of variables in the optimal solution. This matrix is obtained by multiplying A^{-1} with matrix b.

To obtain the inverse of matrix A, shown in Figure 7, begin by arranging matrix A with a coefficient value in each cell. Next, select a square area of the sheet with the same number of cells as matrix A. Enter formula "{=MINVERSE(". Select matrix A and close the parentheses to display a formula that looks like this "{=MINVERSE(B13:F17)}", where the array B13:F17 is matrix A. Press 'Ctrl-Shift-Enter' to display the matrix A^{-1}.

To obtain matrix x*, shown in Figure 7, select a column of cells with the same number of cells as matrix b. Type in the formula "{=MMULT(" then select matrix A, insert a comma, select matrix b and close brackets.

The equation should look like this:

"{=MMULT(B19:F23, B25:B29)}", where the first array is matrix A^{-1} and the second is matrix b. Hit the 'Ctrl-Shift-Enter' key to display matrix x*.

As in the Solver method, profit and cost terms may be discounted to the present by dividing the terms in the equations by the factor: $(1+r)^t$. This changes the coefficients in the analytical solution to produce results that closely match those obtained by Solver as displayed in Figure 4 below.

Matrix A	Eo	E1	Lambda0	Lambda1	Seop0
eq1	0.5	0	1	0	0
eq2	0	0.47619	0	0.95238	0
eq3	0	0	-1	0.95238	0
eq4	1	0	0	0	1
eq5	0	-0.95238	0	0	0.95238
A^{-1}	1.0244	-1.02439	1.02439	0.4878	-0.5122
	-1.024	1.02439	-1.0244	0.5122	-0.5378
	0.4878	0.512195	-0.5122	-0.2439	0.2561
	0.5122	0.537805	0.5122	-0.2561	0.2689
	-1.024	1.02439	-1.0244	0.5122	0.5122
Matrix B	7		Matrix x*	E0*	10.0976
	6.6667			E1*	9.90244
	0			lambda0*	1.95122
	20			Lambda1*	2.04878
	0			Seop0*	9.90244

***Figure 8**: Matrices implementing the analytical solution for 2 periods. Values discounted to the present.*

This matrix multiplication approach also becomes cumbersome when the number of periods under consideration increases. For example, considering 4 periods (as in the next sheet) produces a system of 11 equations that must be solved to arrive at the optimal solution. The optimalextraction array function introduced here solves the system of equations behind the scenes and displays a solution.

The optimalextraction array function uses specific inputs to output the optimal solution for E_i and $lambda_0$ for a PC firm with any number of time periods. The function is input as follows. After selecting the region in which we would like the solution to appear, in this case M8:M9, the formula is entered as follows "= optimalextraction(". Now initial S, interest rate, price, and cost coefficients are selected, separated by commas. Parentheses are closed. The equation should read "=optimalextraction(B2,B3,B8:B9,C8:D9)" for the example above. Now clicking Ctrl+Shift+Enter displays the solution. The very last entry in the analytical column provides the value for lambda, that will be discussed in more detail in a later section.

	A	B	C	D	E	F	G	H	I	J	K	L	M
1													
2	Initial S	20	beginning stock of resource										
3	r	5%	interest rate										
4													
5						Total extractio		Sum Rev	Sum Cost	Sum Profit	Sum PV Profit		
6						20		140	60.003	79.997	78.0979		
7	Time	Price	c0	c1	Stock (BOP)	E	Stock (EOP)	Revenue	Cost	Profit	PV Profit		Analytical
8	0	7	0.25	0.5	20	10.073171	9.9268	70.5122	30.404	40.108	40.1084		10.0731707
9	1	7	0.25	0.5	9.9268	9.9268293	0	69.4878	29.599	39.889	37.9894		9.92682927
10													1.46341463

Figure 9: *Column M, cells M8:M9 display the solutions from the optimalextraction function. M10 displays a value for lambda, a shadow price discussed later.*

Note here that Excel's Solver and the analytical method arrive at almost the same solution but differ in the decimal values. Which one is exactly right? The answer is unfortunately that neither is exactly right. The analytical method through the optimalextraction function calculates the solution the correct way, but given the finite memory of the computer, may produce a slightly imprecise decimal value for the answer.

Excel's Solver is a numerical optimizer that experiments with different values of the endogenous variables to arrive at the optimum solution, which is often very close but not exact. The amount by which Solver's solution may be incorrect is defined by the preset "convergence tolerance" in the program. This tolerance is a range of values clustered around the optimum which Solver judges good enough to stop experimenting with more numerical values. So while the true optimal may be 14, Solver may display

13.999999998. We may be tempted to think that those decimals indicate greater accuracy in the solution obtained when it just means that Solver converged to a solution that it judged acceptably close to the true optimum.

2.1 Isoprofit Mappings

The *PC 2 Period* sheet introduces isoprofit mappings which can be used to understand the implications of changing stocks or interest rates on the present value of profit and therefore intertemporal extraction choices. The Figure below displays these mappings.

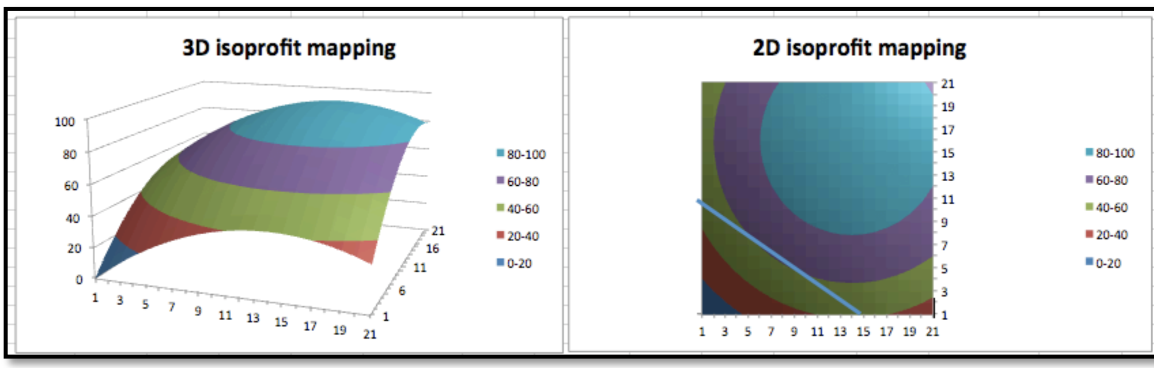

Figure 10: *Isoprofit mappings like those above can be used to understand the discounting of future profits and therefore the intertemporal extraction choices.*

In the 3 dimensional (3D) isoprofit mapping, on the left, the vertical axis represents levels of profit corresponding to an extraction path encompassing extraction amounts in each of the two periods. The 2D isoprofit mapping is a top-down view of the same graph, where the concentric bands represent higher levels of profit. The horizontal axis marks extraction in the first period (time=0), while the vertical axis marks extraction in the second time period (time=1). The diagonal blue line represents the resource constraint. A firm can feasibly produce at any point that line or in the area enclosed by the line, x and y axes.

By changing the interest rate, we can see the intertemporal trade-off in practice. When interest rates rise, extraction in the present becomes more lucrative than extraction in the future. This is illustrated in more detail in the sheet titled *Monopoly2 period, Optimal t.*

2.2 Optimal Extraction Function

In the *PC 2 Period* sheet we saw the analytical solution implemented using the optimalextraction function. This section examines this function in some detail to determine its utility and shortfalls.

The optimalextraction function is constructed using Microsoft Excel's Visual Basic programming language. The initial stock, interest rate, and number of time periods can be varied in the problem and the function applied each time to determine the optimal extraction path under the new parameters. This user-defined function (UDF) can be used to understand the effect of changing parameters on the extraction path and thereby model real geological, economic or political conditions that change.

The optimalextraction function is used by specifying values of P, c_1 and c_2 for each of the time periods in the chosen time frame. Once the parameters are specified, a column of cells is selected in which the number of cells equals the number of time periods under consideration plus one. Once the function is entered here, this column will display the optimal extraction path, containing an optimal extraction value for each period and a value for lambda*, indicating the sensitivity of the stock constraint.

The formula is entered in the form,
"=OptimalExtraction(S-initial, r, Price range, Coefficients range)"
and Ctrl+Shift+Enter is typed to display the solution.
For a given value of initial stock and interest rate, and assuming that stock is completely depleted, the optimal extraction path must span a certain minimum time period.

<u>Limitations of the Optimal Extraction function</u>

Occasionally the optimalextraction function produces an error. This indicates that the true solution is an interior solution (a case for Kuhn Tucker conditions), but the function forces a boundary solution to the problem. A boundary solution is one where a critical point (potential maximum or minimum) lies on the boundary of feasible area of solutions. That is, the function forces the solution to fully exhaust the resource even though the optimal solution requires that some of the resource remain untouched. Unfortunately, the optimal extraction function is unable to show us interior solutions and we must rely on Excel's Solver results for those situations.

2.3 PC 4 Period

The *PC 4 Period* sheet expands the PC 2 period case to 4 periods. Note how the number of equations involved has increased such that the matrices A, B and x are all bigger. Solving the problem manually with 100 time periods would be an exponentially more tedious process.

Matrix A	E0	E1	E2	E3	λ0	λ1	λ2	λ3	Seop0	Seop1	Seop2		Matrix B	
eq 1	0.5	0	0	0	1	0	0	0	0	0	0		7	
eq2	0	0.4761905	0	0	0	0.952380952	0	0	0	0	0		6.6666667	
eq3	0	0	0.4535147	0	0	0	0.907029	0	0	0	0		6.3492063	
eq4	0	0	0	0.431918799	0	0	0	0.863838	0	0	0		6.0468632	
eq5	1	0	0	0	0	0	0	0	1	0	0		20	
eq6	0	-0.952381	0	0	0	0	0	0	0.952381	-0.95238	0		0	
eq7	0	0	-0.907029	0	0	0	0	0	0	0.907029	-0.90703		0	
eq8	0	0	0	-0.8638376	0	0	0	0	0	0	0.863838		0	
eq9	0	0	0	0	-1	0.952380952	0	0	0	0	0		0	
eq10	0	0	0	0	0	-0.952380952	0.907029	0	0	0	0		0	
eq11	0	0	0	0	0	0	-0.90703	0.863838	0	0	0		0	
A^{-1}													Matrix x*	
	1.535976	-0.487225	-0.511586	-0.5371654	0.232011833	-0.243612424	-0.25579	-0.26858	1.535976	1.048751	0.537165		E0*	5.647574
	-0.48722	1.5884139	-0.537165	-0.56402367	0.243612424	-0.255793045	-0.26858	-0.28201	-0.48722	1.101189	0.564024		E1*	5.229953
	-0.51159	-0.537165	1.6409763	-0.59222485	0.255793045	-0.268582698	-0.28201	-0.29611	-0.51159	-1.04875	0.592225		E2*	4.79145
	-0.53717	-0.564024	-0.592225	1.693413909	0.268582698	-0.282011833	-0.29611	-0.31092	-0.53717	-1.10119	-1.69341		E3*	4.331023
	0.232012	0.2436124	0.255793	0.268582698	-0.116005916	0.121806212	0.127897	0.134291	-0.76799	-0.52438	-0.26858		λ0*	4.176213
	0.243612	0.255793	0.2685827	0.282011833	-0.121806212	0.127896523	0.134291	0.141006	0.243612	-0.55059	-0.28201		λ1*	4.385024
	0.255793	0.2685827	0.2820118	0.296112424	-0.127896523	0.134291349	0.141006	0.148056	0.255793	0.524376	-0.29611		λ2*	4.604275
	0.268583	0.2820118	0.2961124	0.310918045	-0.134291349	0.141005916	0.148056	0.155459	0.268583	0.550595	0.846707		λ3*	4.834489
	-1.53598	0.4872248	0.5115861	0.537165395	0.767988167	0.243612424	0.255793	0.268583	-1.53598	-1.04875	-0.53717		Seop1*	14.35243
	-1.04875	-1.101189	1.0487515	1.101189061	0.524375743	-0.55059453	0.524376	0.550595	-1.04875	-2.14994	-1.10119		Seop2*	9.122473
	-0.53717	-0.564024	-0.592225	1.693413909	0.268582698	-0.282011833	-0.29611	0.846707	-0.53717	-1.10119	-1.69341		Seop3*	4.331023

Figure 11: *Matrices used to solve the PC 4 Period problem.*

2.4 Endogenous Price

Basic theories of industrial organization show that a perfectly competitive market, where profit maximizing output is determined at the point where P=MC (Price=Marginal cost) is a special case of the general profit maximizing condition that MR=MC (Marginal revenue=Marginal cost). In the case of perfect competition, the demand curve is horizontal, indicating that no single producer can affect the market price. Here prices are exogenous. From the perspective of an individual producer, demand for the product is infinitely large, at the market price. In this case, the horizontal demand curve also traces the price and marginal revenue, which is constant. This is illustrated in Figure 12 below.

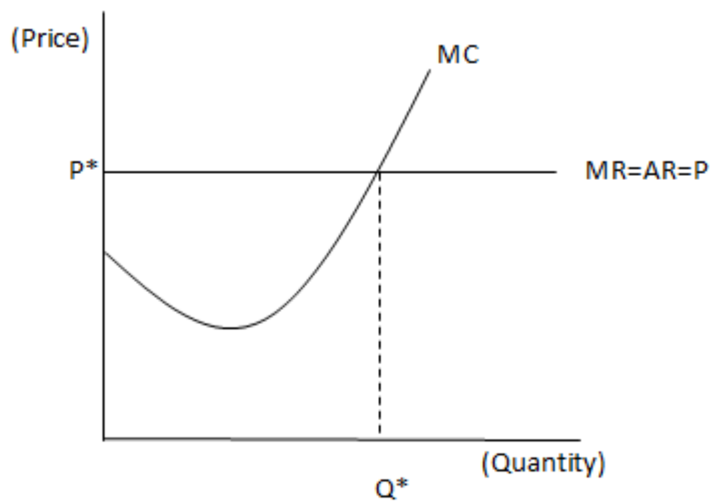

Figure 12: *A firm in a perfectly competitive industry produces Q* corresponding to price=marginal cost, P=MC.*

The discussion of the perfectly competitive (PC) case served as an introduction to the concepts and optimization techniques that are used to solve the extraction problem. However, it does not match the reality of the rare earths industry. China controls approximately 97% of rare earth extractive capacity, and the while some mining companies are privately owned, they are tacitly controlled by the government (Tse 2011). In addition the government has explicit control over REE production through regulation and in recent months has begun to buy private mining companies to create a state owned oligopoly (Bradsher 2011). The rare earth industry in China can therefore be characterized as a monopoly. In this section, the optimal extraction of the resource will be determined for the case of a monopoly in the rare earth market.

In a monopolized industry, the monopolist firm's level of production can influence price. Therefore, prices are endogenous and the relationship between extraction and price is captured by the downward sloping demand curve. In such a situation, price is no longer equal to marginal revenue as in the specific case of perfect competition but is, as always, equal to average revenue. However, the profit maximizing condition of MR=MC remains unchanged. Intuitively this means that when MR>MC production must increase, and when MR<MC production must decrease to maximize profit. The monopoly output decision is illustrated in Figure 13 below.

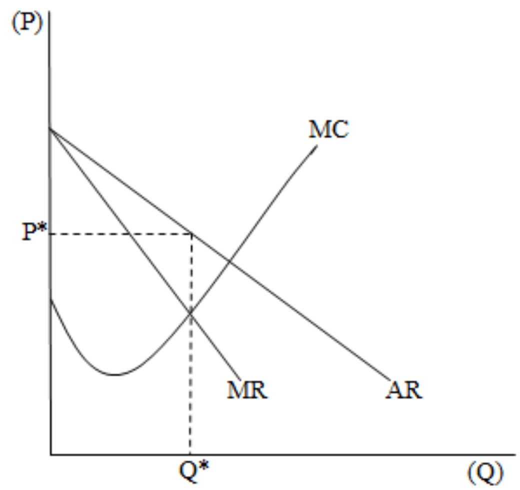

Figure 13: *A monopoly firm produces Q* corresponding to marginal revenue=marginal cost, MR=MC.*

In summary, the salient differences between a perfectly competitive extractive industry and monopoly extractive industry are as follows:
(i) The monopolist faces a downward sloping demand curve unlike the horizontal and infinitely elastic demand curve of the perfectly competitive firm.
(ii) In the monopoly case, the extraction and output of a firm determine market prices. In the perfectly competitive case, no individual firm can affect market prices. Firms in this industry are price-takers.
(iii) A monopoly firm controls all of N number of resource sources. In the context of rare earth elements, these N sources would be mines, each potentially extracting a different ore with different extraction costs. For the purposes of the discussion of monopoly

extraction in this paper, the existence of N number of sources will be ignored. A single cost function is used to describe costs of extraction for all ores containing rare earth elements.

2.5 Monopoly 2 Period

The *Monopoly 2 Period* sheet models the extraction problem as a monopoly with endogenous prices.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1																
2																
3																
4	Initial	20.000														
5	r	5%														
6																
7							Total extractio		Sum Rev	Sum Cost	Sum Profit	Sum PV Profit				
8		$P = d_0 - d_1E$		$TC = c_0E^2 + c_1E$			20.000		499.983	60.004	439.979	429.545				
9	Time	d0	d1	c0	c1	Stock (BOP)	E	Stock (EOP)	Revenue	Cost	Profit	PV Profit	P*		Analytical	P*
10	0.000	35.000	1.000	0.250	0.500	20.000	10.093	9.907	251.382	30.512	220.870	220.870	24.907		10.093	24.907
11	1.000	35.000	1.000	0.250	0.500	9.907	9.907	0.000	248.601	29.492	219.109	208.675	25.093		9.907	25.093
12															9.268	

Figure 14: *The setup for a monopoly extraction problem. Note columns B and C that contain parameters describing the demand curve.*

The downward sloping demand function for the monopoly case is represented as:

$$P_i = d_{0i} - d_{1i}E_i$$

where d_0 and d_1 are coefficients indicating the intercept and slope of the demand function. The quadratic cost function of the form, $TC = c_{1i}E_i^2 + c_{2i}E_i$ is used. The unconstrained optimum solution can be obtained by maximizing the profit value of the resource based on the demand and cost functions available.

$$\pi = d_{0i}E - d_{1i}E_i^2 - c_{1i}E_i^2 - c_{2i}E_i$$

$$\frac{d\pi}{dE_i} = d_{0i} - 2d_{1i}E_i - 2c_{1i}E_i - c_{2i} = 0$$

$$E^* = \frac{d_{0i} - c_{2i}}{2(d_{1i} - c_{1i})}$$

However, the unconstrained optimum only serves as an upper bound on extraction in each period. The constrained problem is more realistic since it incorporates information about the resource constraint. Since rare earth elements are depletable and exist in finite quantity, it is possible that the unconstrained optimal solution cannot be reached simply because of resource limitations. On the other hand, the optimal solution may lie within the constraint in which case, the scarcity of the resource would not constrain the optimal extraction path.

The constrained optimization problem is described below. The Lagrangian used here incorporates the stock constraint and differs from the case of perfect competition since it includes a downward sloping demand curve.

$$L = \sum_{i=0}^{n} \frac{(PE_i - TC_i)}{(1+r)^i} - \frac{\lambda_i (S_{EOPi} - E_i - S_{BOPi})}{(1+r)^i}$$

$$= \sum_{i=0}^{n} \frac{d_{0i}E - d_{1i}E_i^2 - c_{1i}E_i^2 - c_{2i}E_i}{(1+r)^i} - \frac{\lambda_i (S_{EOPi} - E_i - S_{BOPi})}{(1+r)^i}$$

In this equation ‘i’ indicates the period in which extraction is occurring. While extraction could theoretically occur in infinitely many time periods, the value of extraction in the future would be discounted by a factor containing the interest rate and as a result, the present value of “far off” future extraction would contribute negligibly to the present value of profit. In reality therefore, a rational actor would not consider their resource extraction path in the “far off” future. Though, it is unclear where the “far off” future begins, and PV profit is so insignificant that it does not matter.

To solve the optimization problem, the Lagrangian is differentiated with respect to each of the variables. The resulting system of equations is solved using the matrix multiplication method to find the optimal extraction E* in each time period and the value of the Lagrangian multiplier. The value of the Lagrangian multiplier indicates the tightness of the constraint.

The optimalextractionM function appears in this Excel sheet. This function can solve multi-period extraction problems given the monopoly conditions of downward sloping demand. The particular function in the spreadsheet contains a downward sloping demand curve of the kind used in the Lagrangian above. This linear demand curve has a slope term d_1 and intercept term d_0. The values of the slope and intercept terms could be changed to model a demand curve that is more or less steep.

The implication of the downward sloping demand curve is that demand is negatively related to price. Higher prices lead to lower demand and vice-versa. In this case, it also means that producers' extraction decisions are linked to demand through the price mechanism. Therefore, a monopolist seeking to exploit their monopoly power in the market may find that with very little extraction and therefore very high prices, demand may shift to a backstop technology or other substitute. Additionally, the process of developing substitutes may become economically viable and eventually put the monopolist out of business.

The optimalextractionM function also contains the quadratic cost function that is described in the Lagrangian above. It displays the solution to the system of equations in the form of a vertical array function that contains values for E* (optimal extraction) followed by the value of the Lagrangian multiplier.

The optimalextractionM function is used much like the optimalextraction function from the case of perfect competition. A column of t+1 cells are selected where t is the number of time periods for which optimal extraction is being computed. The formula is then entered in the format,

"=optimalextractionM(S-initial, r, Demand coefficients as range, Cost coefficients as range)".

Ctrl+Shift+Enter is hit to display the solution. The Figure below displays this function as it is used in the Excel sheet for 2 periods. Here, the formula is specifically, "=optimalextractionM(B4, B5, B10:C11, D10:E11)" , where cell B4 contains initial stock, B5 contains the interest rate, the range B10:C11 contains demand coefficients and the range D10:E11 contains cost coefficients.

The results of the optimalextractionM function for extraction in each period must be less than or equal to the solution generated by the unconstrained solution. The unconstrained model determines the level of output that maximizes profit for a given cost and revenue function. The point that it determines as optimal is that at which marginal review (MR) is equal to marginal cost (MC), MR=MC. For any point at which MR<MC, profit will be increased by decreasing production until MR=MC. Similarly, for any point MR>MC, profit will be increased by increasing production until MR=MC. Therefore, the extraction values for each period of the monopoly optimal extraction path cannot be more than the optimal extraction in the unconstrained model because this would represent a case of MR>MC or overproduction, where profits would be increased by reducing production. However, production could be less than the unconstrained optimal solution since a limited amount of the resource may make it impossible for production to reach the unconstrained optimal solution.

The result obtained in the monopoly model is computationally similar to the perfectly competitive situation since it follows the same logic and similar mechanics of optimization. In both cases the product of price and extraction computes revenue while in the monopoly case this product can be unpacked further to see the effect of demand coefficients d_0 and d_1. Barring this minor difference the remainder of the constrained optimization problem is identical under the two market structures.

This is easier seen in the profit maximization condition of the two market structures. In both cases optimum extraction is reached when MC=MR. Differences only arise in the determination of prices.

In this monopoly model, prices are determined by monopoly extraction and modeled in the demand function. Optimal extraction is determined using the demand coefficients d_0 and d_1 after which price in each period is calculated by plugging in the corresponding value for optimal extraction into the demand function: $P_i = d_{0i} - d_{1i}E_i^*$. Prices appear in column P of the Excel sheet.

2.6 Effect of the Interest Rate on Extraction Paths

In the *Monopoly 2 period* sheet, the effect on interest rate on the intertemporal extraction choices is examined again. In particular, we notice that with higher interest rates, the isoprofit graph shrinks and some extraction is shifted to the first time period.

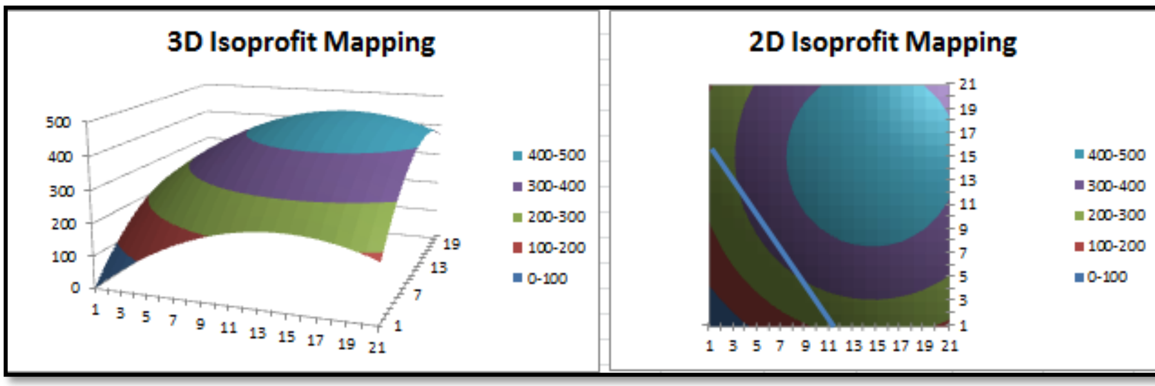

Figure 15: *Isoprofit graphs at 5% interest rate.*

At an interest rate of 5000%, the isoprofit bands in the 2D isprofit mapping are almost vertical, indicating that the value of extraction in the second period is discounted so heavily as to be negligible.

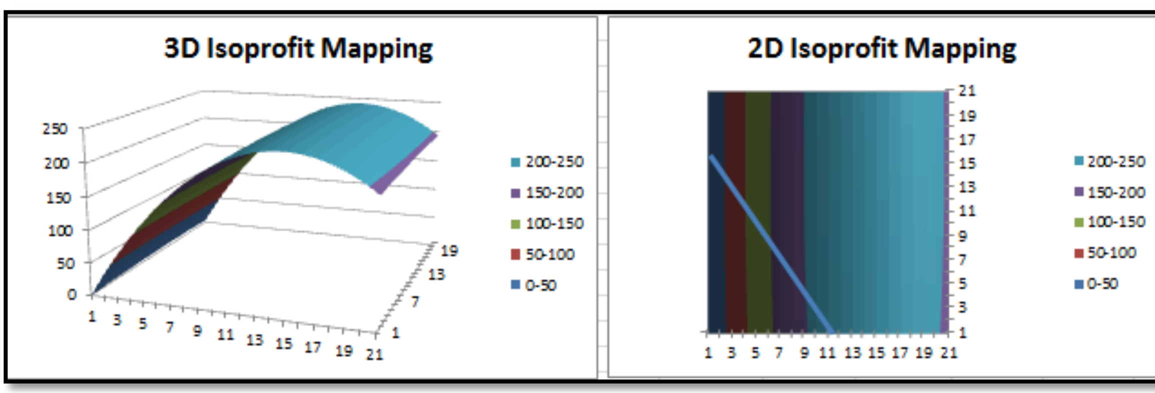

Figure 16: *Isoprofit graphs at 5000% interest rate.*

As a result, the monopolist moves to higher isoprofit bands by moving horizontally across the graph. This is achieved by increasing extraction in the first time period.

The next question then is how much the monopolist increases their extraction in period one. Will a monopolist completely exhaust the resource in the first time period? No, the monopolist is bound by the profit maximizing calculus described earlier. Extraction in each period will be no more than the unconstrained optimum in each period. With the demand and cost parameters chosen here, this unconstrained optimum is 14 units. Solver arrives at this solution to produce 14 units in each of the time periods, displayed in cells G10:G11.

The analytical solution and prices displayed here are wrong. This is because the OptimalExtractionM function assumes that the resource must be fully extracted. It therefore cannot generate a solution that leaves a positive end of period stock. In this case therefore it produces wrong values for extraction in each period.

Returning to the Solver solution, note that while equal units are produced in each of these time periods, the PV profit from E0 is 245 while that from E1 is 4.8. Clearly extraction in the second period has little value. Extraction in subsequent periods, while not displayed here are known to be of drastically smaller values. It is therefore likely that some fraction of the resource may not ever be extracted. The Kuhn Tucker conditions and Solver allow us to arrive at this solution. That is illustrated graphically below.

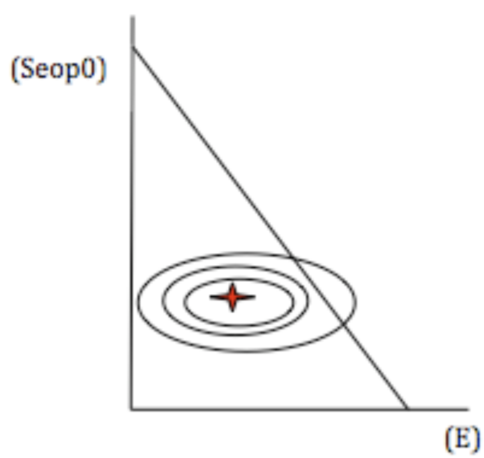

Figure 17: The optimal solution is marked in red. The diagonal line represents the points along which the resource is completely exhausted.

In the Figure above the optimal solution is marked in red while the diagonal line represents the points along which the resource is fully extracted. The concentric rings are isoprofit curves that make up the 3-D graphs from Figures 15 and 16. When the interest rate is raised, the isoprofit curves and unconstrained optimum move closer to the resource constraint and, as in the Figure above might actually move within the space enclosed by the resource constraint. This situation would be one in which it would be optimal not to fully exhaust the resource.

2.7 Monopoly 100 period

The *Monopoly 100 Period* sheet extends the earlier discussion of monopoly extraction choices to 100 periods. It then graphs price and extraction paths to better understand the effects of time on optimal extraction and price paths. The choice of 100 years in this example was arbitrary. The advantage from using such a long period of time is that the long-run effects of discounting profit can be seen in the extraction path, pictured below.

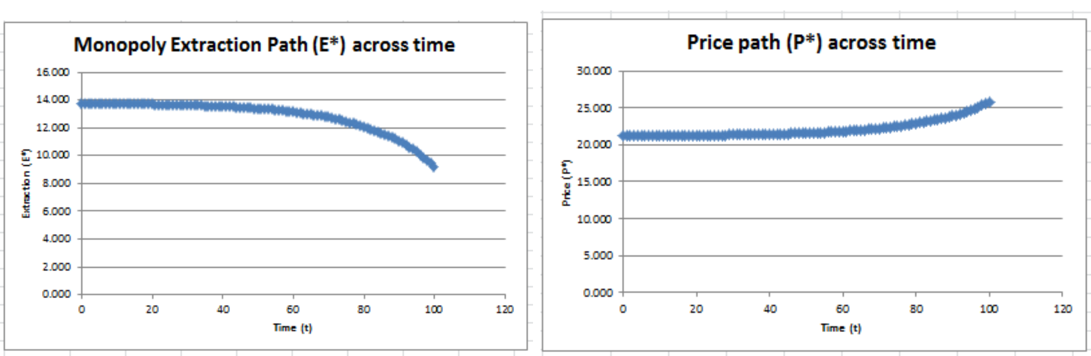

Figure 18: *The monopoly extraction path slopes downward while the price path slopes upwards.*

There are two things to note here. First, extraction path across time is downward sloping and the price path is upward sloping. This appears counter-intuitive but has an explanation. The extraction path is downward sloping because the monopolist faces scarcity and a positive interest rate. In the first few periods, production is very close to the unconstrained optimal solution. Then it begins to decrease when the present value of profit is discounted more and more drastically. By period 100 the present value of profit from the period is 1.6 (cell L109), down from 238 in the first period. While these numbers are arbitrary and arise from the parameter choices made, they illustrate the effect of discounting profit over time. A result of the discounting is that, ceteris paribus, it is optimal to extract more units of the resource in earlier periods than later periods, however, extraction in no period will ever exceed the unconstrained optimum because every unit of production in excess of the optimal would erode total profits and violate the profit maximization condition.

The extraction we are considering in this workbook concerns depletable resources, that cannot be renewed in any time frame relevant to economic decisions. As a result a monopolist is able to earn scarcity rents from the resource. Scarcity rents are at the core of optimal extraction, and what make the problem really interesting from an economic perspective. Also known as the marginal user cost of a resource (Sweeney 1993), this scarcity rent is a form of opportunity cost that is implicit in the rising marginal cost of extraction across time (Coats et al 2009).

In general, an opportunity cost is defined as the cost incurred from the forgoing the next best alternative to a good. In this case, because the resource is finite, extracting it today means that it will not be available for extraction tomorrow. As less and less of the resource remains, or as end-of-period stock declines, the opportunity cost of extracting rises because each unit of the resource is more valuable. Also, an amount x extracted today is worth more than the same amount extracted tomorrow, because a positive interest rate discounts its value over time. Therefore, the opportunity cost of waiting to extract a unit of the resource is higher, the longer we wait.

In summary, two forces cause rising marginal costs of extraction. These are:
i) Opportunity cost of scarcity – The less there is, the higher the opportunity cost of extracting each unit. This is the forgone value of extracting the resource in the future.
ii) Opportunity cost of waiting – Because future profits are discounted to the present, it is more profitable to extract any amount x of the resource than wait to extract that x in the future. The longer we wait, the higher that opportunity cost.

Scarcity rents or marginal user costs are implicit or hidden because they do not appear as a separate expression in the cost functions used here. Instead, they are the embodied in the effect of extraction (E) on the marginal cost function.

$$TC = c_0 E_i^2 + c_1 E_i$$
$$MC = 2c_0 E_i + c_1$$

Because marginal costs are on the rise, extraction falls gradually to zero and may or may not fully extract the resource (Tietenberg 2000, 133).

Assuming that demand remains constant over time, price must rise with marginal user cost, because a monopolist will only extract in the future if the discounting is offset by some increase in price. It happens that in equilibrium for a monopoly, this price must rise specifically at the rate of interest (Coats et. al 2009). While this is an interesting result, it will not be explored further in this paper because it requires the development of the market equilibrium analysis of the extraction industry.

The analytical solution is correct as presented here, but we can create a solution in which it generates a different problem from those explored earlier. In cell B3 of the Excel sheet, change the value of initial S to 1000 units from 1300 and then observe the solution presented by the optimalextractionm function in column O. Starting in cell 0103, this solution suggests that extraction should be negative. Since this is impossible, it is clearly wrong.

	J	K	L	M	N	O	P
1							
2							
3							
4							
5							
6	**Sum Cost**	**Sum Profit**	**Sum PV Profit**				
7	4866.014	23769.931	4958.833				
8	**Cost**	**Profit**	**PV Profit**	**P***		**Analytical**	**P***
94	39.654	232.185	3.671	23.366		4.714	30.286
95	39.050	231.654	3.488	23.462		4.260	30.740
96	38.423	231.073	3.313	23.562		3.783	31.217
97	37.765	230.432	3.147	23.669		3.282	31.718
98	37.066	229.713	2.988	23.783		2.756	32.244
99	36.317	228.898	2.835	23.906		2.204	32.796
100	35.505	227.962	2.689	24.041		1.624	33.376
101	34.620	226.876	2.549	24.190		1.015	33.985
102	33.621	225.566	2.414	24.360		0.376	34.624
103	32.608	224.142	2.284	24.536		-0.295	35.295
104	31.550	222.548	2.160	24.722		-1.000	36.000
105	30.453	220.770	2.041	24.918		-1.740	36.740
106	29.321	218.801	1.926	25.124		-2.517	37.517
107	28.163	216.633	1.816	25.339		-3.333	38.333
108	26.985	214.260	1.711	25.563		-4.190	39.190
109	25.797	211.682	1.610	25.793		-5.089	40.089
110						0.359	
111							

Figure 19: *The analytical solution highlighted here is wrong because it displays negative values for extraction.*

The reason this occurs is because, in addition to needing to exhaust the resource, the analytical solution also cannot display solutions in which extraction in any time period is zero. Now run Solver and note that it is optimal to end extraction after the 92^{nd} period. When the analytical solution runs into problems, it displays an error. The specific error says that the Lagrangian is forcing the solution to be on the constraint. In a two period case this would look like the graph in Figure 20.

In the Figure, the unconstrained optimal (marked red) lies outside the constraint, the constrained optimum(blue) lies at E_0 on the x-axis (where E_1=0) but the Lagrangian forces a solution(black) in which the E_0 is positive but E_1 is negative. The reason that the Lagrangian produces its negative value of E_1 is because it cannot incorporate the constraints $E_0 \geq 0$ or $E_1 \geq 0$.

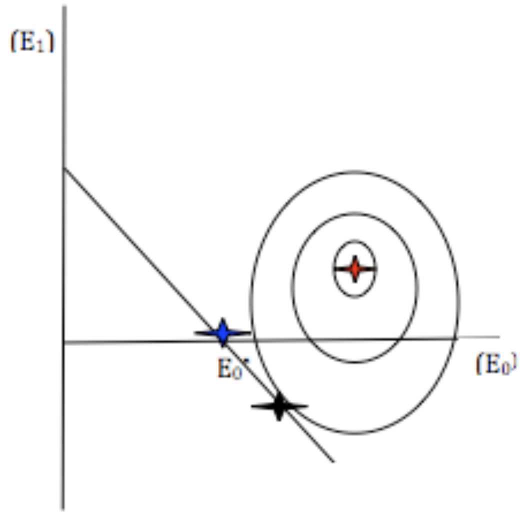

Figure 20: *The unconstrained optimum in red is unreachable because stock is limited. The constrained optimum in blue lies on the x-axis. The Lagrangian produces the wrong solution marked in black.*

Expanding from the two-period case displayed above, where E_1*=0, in the particular example demonstrated in Excel, a firm maximizes the present value of the resource by exhausting the resource in 92 periods. Therefore extraction is zero in subsequent periods.

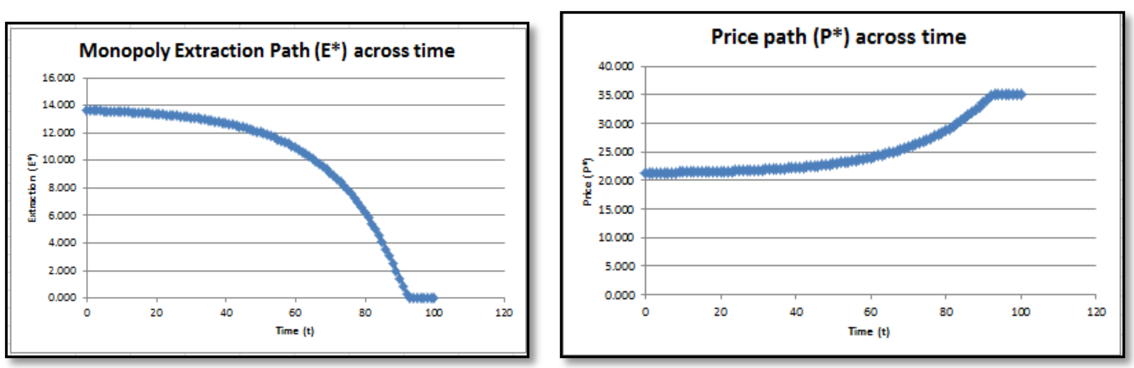

Figure 21*: The correct solution requires extraction to be complete in 92 periods. Extraction path and price path still slope downward and upward respectively.*

The extraction and price trajectories change when initial stock is changed, but they also change when interest rate changes as discussed in section *2.7 Monopoly 2 Period.* When interest rate is increased, the extraction path across time is steeper. That is, extraction reduces more rapidly over time because the value of profit from later periods is discounted more heavily. This is discussed more fully in the Excel sheet *r Comparative Statics.*

2.8 PC Constant Price

This sheet illustrates a perfectly competitive (PC) extraction industry where prices, stock and interest rate are exogenous. Here we maintain price at a constant level and set cost parameters such that marginal costs is rising and the optimal extraction values are high enough that the constraint is binding.

Occasionally, it may make sense to hold prices constant. When prices in the industry do not fluctuate much or only fluctuate from random shocks might be one such case (Sweeney 1993). It is displayed here. In reality, the recent past has seen sharp increases and subsequent reductions in the prices of rare earth elements for a variety of factors relating both to economic scarcity and policy decisions. Nonetheless, for the purposes of the theoretical discussion, this sheet is useful.

4	**Initial S**	43000000.000	beginning stock of resource									
5	**r**	0.050	interest rate									
6												
7							**Total Extraction**	**Unextracted Resource**	**Sum Rev**	**Sum Cost**	**Sum Profit**	**Sum PV Profit**
8			**TC = $c_1E^2 + c_2E$**				43000000.03	0	3.61157E+11	1.43275E+11	2.17882E+11	46144485670
9	**Time**	**Price**	**c1**	**c2**	**MC**	**Stock (BOP)**	**E**	**Stock (EOP)**	**Revenue**	**Cost**	**Profit**	**PV Profit**
10	0	**6110.000**	0.005	2.000	732.000	43000000.000	**73000.000**	42927000.000	446030000.000	26791000.000	419239000.000	419239000.000
11	1	**5330.000**	0.005	2.000	732.000	42927000.000	**73000.000**	42854000.000	389090000.000	26791000.000	362299000.000	345046666.667
12	2	**6800.000**	0.005	2.000	882.000	42854000.000	**88000.000**	42766000.000	598400000.000	38896000.000	559504000.000	507486621.315
13	3	**5450.000**	0.005	2.000	922.000	42766000.000	**92000.000**	42674000.000	501400000.000	42504000.000	458896000.000	396411618.616
14	4	**7410.000**	0.005	2.000	952.000	42674000.000	**95000.000**	42579000.000	703950000.000	45315000.000	658635000.000	541860644.485
15	5	**5500.000**	0.005	2.000	1192.000	42579000.000	**119000.000**	42460000.000	654500000.000	71043000.000	583457000.000	457153826.509
16	6	**3150.000**	0.005	2.000	1192.000	42460000.000	**119000.000**	42341000.000	374850000.000	71043000.000	303807000.000	226705461.006
17	7	**4160.000**	0.005	2.000	1202.000	42341000.000	**120000.000**	42221000.000	499200000.000	72240000.000	426960000.000	303432500.712
18	8	**10300.000**	0.005	2.000	1202.000	42221000.000	**120000.000**	42101000.000	1236000000.000	72240000.000	1163760000.000	787678575.955
19	9	**8450.000**	0.005	2.000	1292.000	42101000.000	**129000.000**	41972000.000	1090050000.000	83463000.000	1006587000.000	648854955.149
20	10	**8450.000**	0.005	2.000	1302.000	41972000.000	**130000.000**	41842000.000	1098500000.000	84760000.000	1013740000.000	622348421.644
21	11	8450.000	0.005	2.000	8145.758	41842000.000	814375.831	41027624.169	6881475771.479	3317668721.793	3563807049.686	2083684172.252

Figure 22: *The setup for constant price in Excel. The values bolded in blue are observed values of price and extraction for China from years 2000-2010.*

In this sheet, the prices and quantity extracted that are bolded in blue are observed values for rare earth oxides from 2000-2010. Subsequently, price is maintained at $8,450 per unit while optimal extraction is determined by Excel's Solver.

The results for extraction across time are plotted in the graph below. With constant prices and a steadily rising discount factor (1+r)^t that is used to discount profit to the present, we see that extraction gradually reduces in subsequent time periods till it eventually falls to zero. The particular rate of reduction or marginal extraction will depend on marginal cost and therefore our calibration, however we see that extraction does not fall abruptly from a very large number to zero, rather it is smoothed out over a period of time.

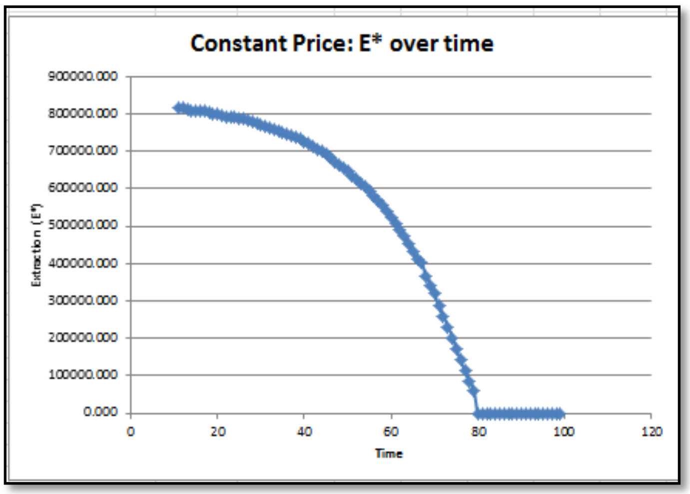

Figure 23: *With constant prices and a certain set of cost parameters, the extraction curve is smooth over time, eventually reaching zero.*

If the resource were so abundant or costs so high that the constraint was not tight, then the solution to the unconstrained version would be a lower value of extraction. In that case, the firm would simply extract the same (small) amount of resource in each period, making hardly a dent in the resource and there would be no discussion of intertemporal extraction choices. We can replicate this situation by setting c1=c2=0.02 and running Solver. Now the graph of E* over time is a straight line.

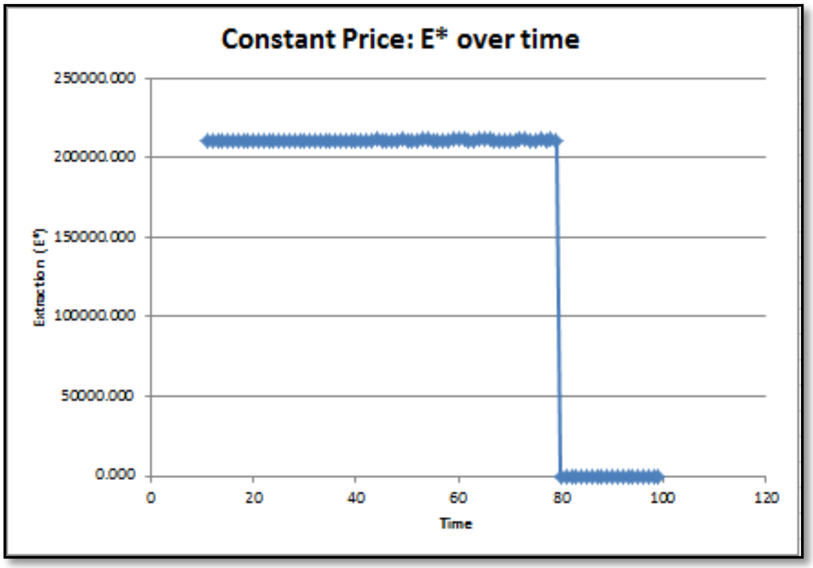

Figure 24: *When the constraint is not tight, intertemporal extraction choices are less relevant. Extraction is constant across time.*

2.9 PC Random Price

In this sheet, the prices and quantity extracted that are bolded in blue are true values for rare earth oxides from 2000-2010. Subsequently, prices are generated randomly by Excel in column M using the function "NVRandomnormal(mean, standard deviation)" where randomly generated prices prices are normally distributed around 5000, with a standard deviation of 500. These randomly generated prices appear in column B, from row 21 onwards.

To generate a new set of random prices type "ctrl+alt+F9". The numbers in column B change but Excel will not automatically recalculate the extraction path. To recalculate the extraction path go to the Data tab and run Solver. The chart automatically updates to reflect the new extraction path for the given set of random prices.

The results for extraction across time are plotted in Figure 25 below. The sharp upward spike occurs at the shift between real historical values of extraction and those generated using trial cost parameters in the Excel sheet. Thereafter, prices bounce around an average of 5000, causing optimal extraction to vary from period to period. The general trend is a downward sloping extraction curve.

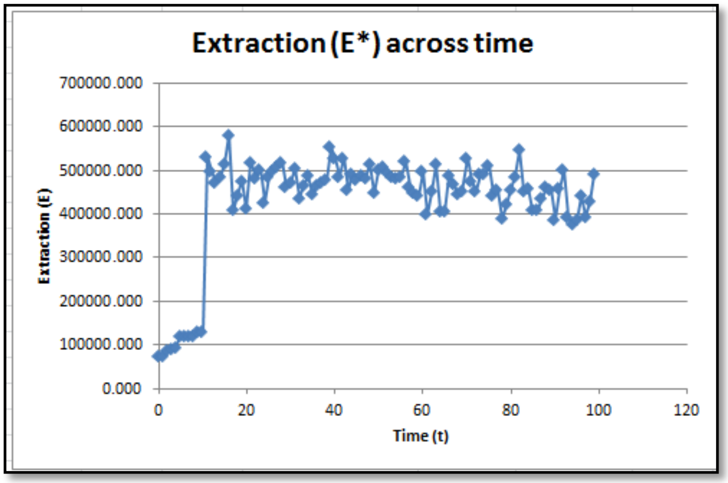

Figure 25: *Extraction across time fluctuates but overall trends downwards.*

The downward sloping trend is more easily visible when we change the standard deviation of the random price generator to 100. In cell B21, the formula is changed to "NVRandomnormal(5000,100)". This formula is then applied to lower cells in the column. Next Solver is run to generate an optimal extraction path corresponding to the new prices.

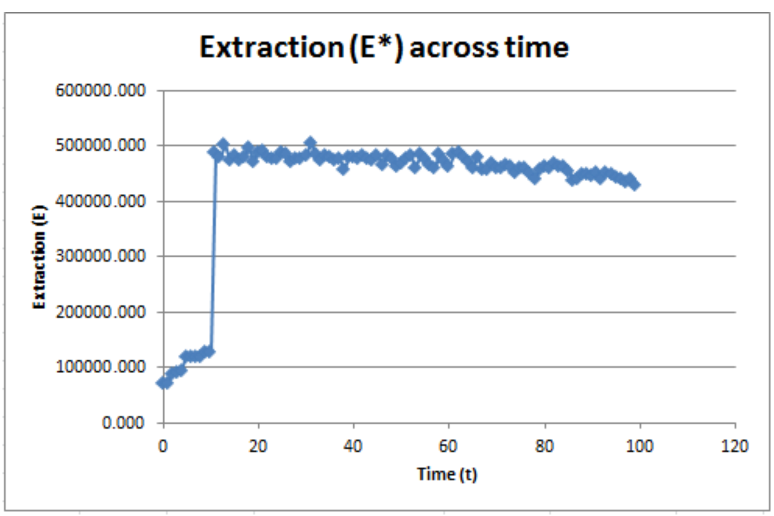

Figure 26: *With the standard deviation of random prices reduced, the gradual decrease in extraction across time is more evident.*

The rationale for generating random prices is that they provide a closer approximation of price changes caused by random market forces that might influence the optimal extraction path. In this example, we see that random prices cause extraction in each period to vary considerably from prior and subsequent periods to create a jagged graph. However, the predominant trend is for extraction to reduce over time.

Extraction might come close to exhausting the resource and generate a steeper (still downward sloping) extraction path if the unconstrained optimum were a larger percentage of the total resource stock. In this case, where the numerical example uses the USGS estimate of China's initial stock, extraction in each period is a very small portion of the total resource. Therefore marginal user cost or scarcity rents accruing from the resource are quite small. The result is that the graph does not display a dramatic reduction in the extraction over time.

2.10 Hotelling's Price

This sheet illustrates a perfectly competitive extraction industry where market equilibrium price rises over time. This result, known as Hotelling's r percent rule can be proved mathematically from modeling market equilibrium in a perfectly competitive setup. It will not be proved here, but instead applied to the model to illustrate the effects of rising prices on optimal extraction.

In market equilibrium in a perfectly competitive market, Hotelling (1931) finds that the opportunity cost of the resource (marginal user cost or scarcity rent) rises at the rate of interest over time (Sweeney 1993) . Since prices under perfect competition are the sum of marginal extraction cost and opportunity cost, a component of prices also rise at the rate of interest. The result is that prices rise, but at a rate less than the interest rate. This price increase offsets the discounting to some degree, but the present value of price falls (Sweeney 1993).

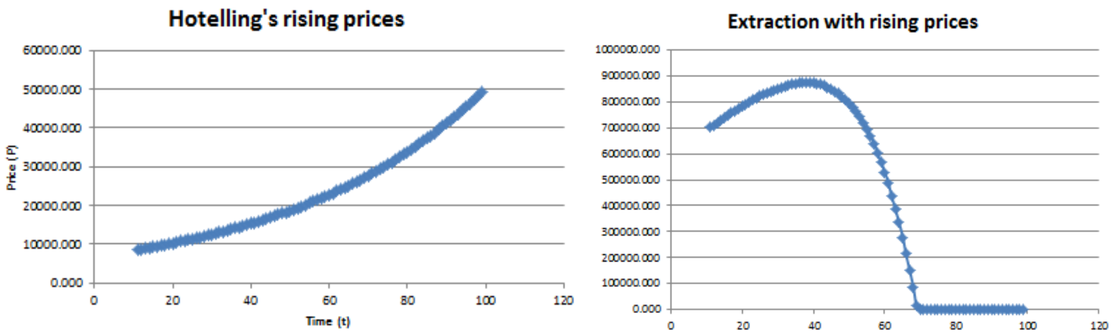

Figure 27: *Prices rise but extraction falls over time.*

Exactly how responsive to these future price increases is extraction in each time period? We can examine this using the price elasticities of extraction, calculated as the ratio of the percentage change in extraction to the percentage change in price.

In this case of a perfectly competitive market with prices rising at a rate less than interest, extraction is less responsive to price changes in later periods and negatively related after some time. The negative relation probably occurs because discounting becomes much larger relative to the price increase in later periods.

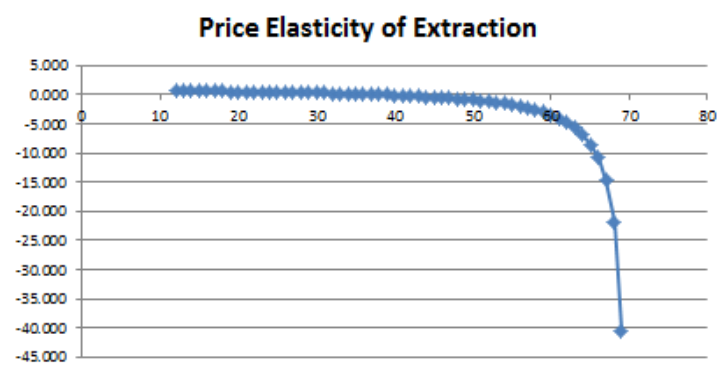

Figure 28: *Price elasticity of extraction becomes more negative over time.*

Price elasticity of extraction

Exactly how responsive to these future price increases is extraction in each time period? We can examine this using the price elasticities of extraction, calculated as the ratio of the percentage change in extraction to the percentage change in price.

$$\varepsilon_{t+1} = \frac{\Delta E}{\Delta P} * \frac{P_t}{E_t}$$

In this case of a perfectly competitive market with prices rising at the rate of interest, elasticities are unresponsive to the time period and price elasticities of extraction are constant.

2.11 Monopoly Price Changes

This sheet considers how monopoly prices might change over time and the effect this has on optimal extraction. In this sheet we demonstrate decreases in the market equilibrium price for a monopoly firm. Because a monopolist has control over prices through extraction decisions, prices do not change exogenously as in the case of perfect competition. However, some parameters such as d0, d1 do change in ways that the monopolist cannot control. In this particular example, d0 decreases at the rate of 1% per year as a result of substitutes entering the market. The result is a demand curve with a constant slope but decreasing intercept. As a result, optimal extraction in each period is lower. The graphs below show that as prices fall, extraction also falls.

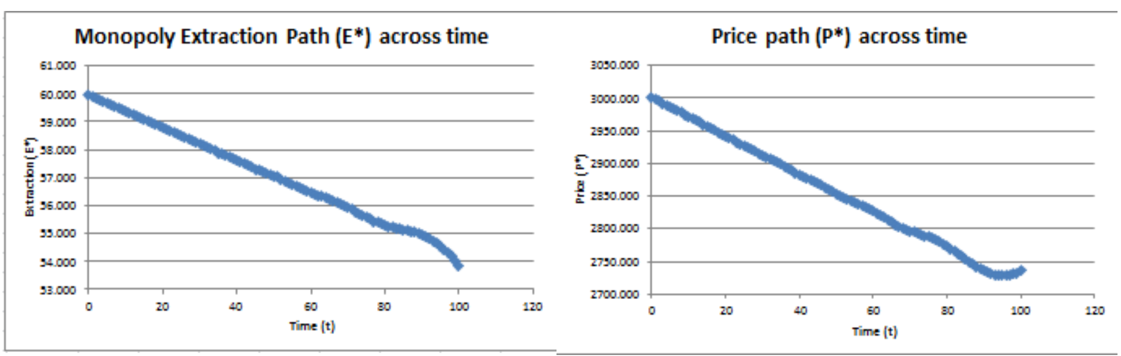

Figure 29: *As demand reduces, both price and the monopoly extraction paths slope downwards.*

2.12 Initial Stock Comparative Statics

In general, the process of changing just one exogenous variable and tracing the response of endogenous variables over time is known as comparative statics. In this sheet, we perform comparative statics on the monopoly model by changing levels of initial stock and chart the extraction paths corresponding to each level of initial stock.

To begin, we need to download the Comparative Statics Wizard add-in to Excel. Go to http://www.depauw.edu/learn/microExcel/MicroBook/CSWiz.htm. Detailed instructions on installing the add-in appear at the website.

Once the add-in is installed, the problem is set up as follows:
- Click on the Add-ins tab in Excel and select Comparative Statics Wizard. For objective cell select L7 which calculates PV profit. Solver will maximize this objective function.
- Next, select the range G20:G109 for endogenous variables. This range contains endogenously determined extraction paths and changes when initial stock is changed.
- Then we select B3:B4 as the range of exogenous variables.
- Clicking 'Next' we are prompted to run Solver.
- Once Solver has finished running once, click 'Next' to go to the section where the comparative statics conditions are specified.
-Here we indicate that cell B3 is the one we would like to change, describe the increments in which we would like to change it, and the number of times we want to shock the system.
- In the example graphed below, initial stock was set at 1,000,000 units and increased by 1,000,000 10 times.

The results of the comparative statics appear in a separate sheet. The results for the particular example described above appear in cells Z8:DM19 of this worksheet. Each row in that range represents an extraction path. These paths are graphed in Figure 30.

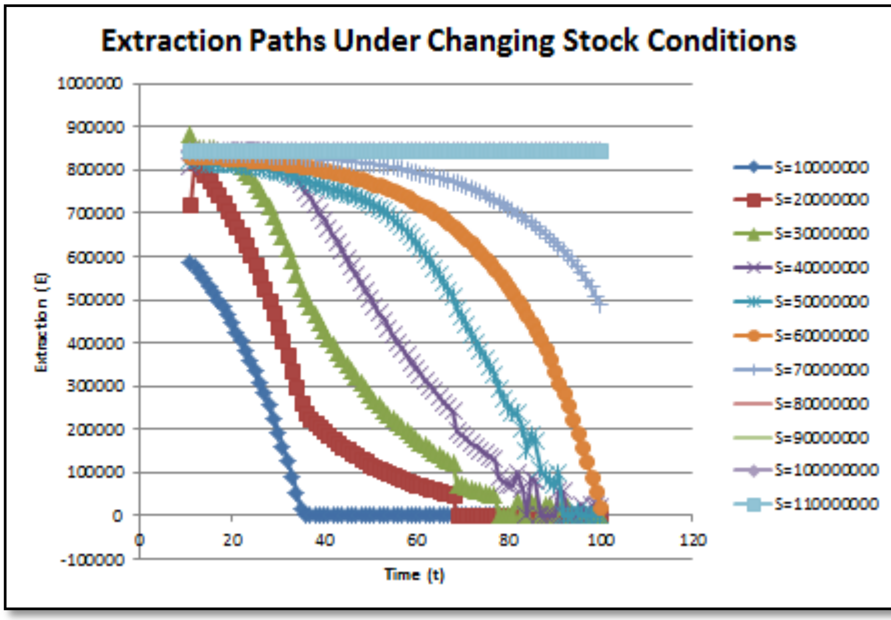

Figure 30: *Extraction paths under different initial stock (S) conditions. The curves move outward and upward with larger initial stocks.*

We note that as initial stock is increased in this manner, the optimal extraction path shifts outward and falls to zero extraction relatively slower for a given interest rate. This indicates that with a larger stock, more is extracted in each time period (as long as extraction remains below the unconstrained optimum) and extraction takes place over a longer period of time.

Theoretically, this pattern makes sense because the marginal opportunity cost of each additional unit of extraction is relatively small when the resource is more abundant. With a larger stock, the opportunity cost of present extraction rises slower relative to extraction paths when the resource is scarcer, and therefore extraction does not reduce as dramatically.

2.13 Interest Rate Comparative Statics

In this sheet we shock the monopoly model with different interest rates and chart the extraction paths corresponding to each level of interest rate.

This problem is set up almost identically to the comparative statics for initial stock shocks.

- L7 is still the objective cell calculating PV profit. Solver will maximize this objective function.
- G20:G109 are selected as endogenous variables. This range contains endogenously determined extraction paths and changes when the interest rate is changed.
- B3:B4 is selected as the range of exogenous variables.
- Once Solver has run once, we set up the comparative statics portion of the Comparative Statics Wizard.
-Here we indicate that cell B4 is the one we would like to change, describe the increments in which we would like to change it, and the number of times we want to shock the system.
- In the example graphed below, initial interest rate was set at 5% (0.05) and increased 10 times by 1% (0.01).
- When the Wizard is run, results appear in a separate data sheet. The results for this particular example are pasted into cells Z8:DM19 of this worksheet. Each row in that range represents an extraction path which is then graphed across time.

We note that as interest rates are increased, the optimal extraction path becomes steeper and falls to zero extraction relatively quickly for a given level of initial stock. This indicates that with higher interest rates, the stock is preferentially extracted in earlier time periods and the resource is entirely exhausted sooner. Figure 31 below charts the results for this particular example.

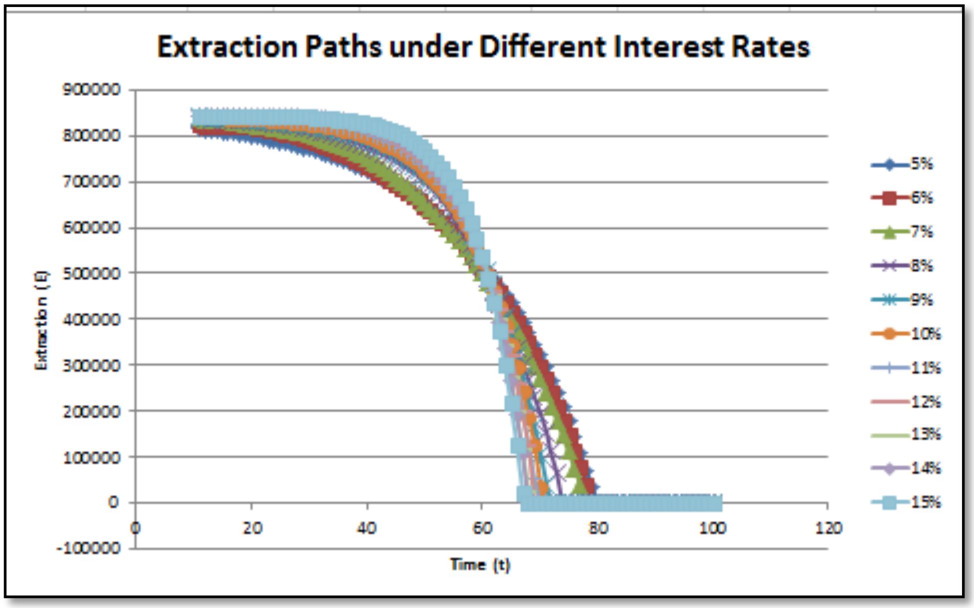

Figure 31: *Extraction paths with different interest rates. At higher interest rates the extraction paths slope downward more steeply.*

We saw this pattern on the Monopoly 2 period sheet where higher interest rates caused isoprofit curves to become steeper. The theoretical basis for this response to higher interest rates is that extraction in future time periods is discounted by a larger factor when interest rates rise and therefore, some extraction moves from those periods into earlier periods where profits are not discounted as heavily.

2.14 Neodymium Prices

This sheet is calibrated with the real prices of neodymium metal from Jan'09 through Jan'12. This price information was obtained from the image below from Bradsher (2011). The specific prices were estimated from that graph using a plot digitizer from http://plotdigitizer.sourceforge.net/.

The purpose of this calibration is to see the effect on intertemporal extraction choices of a real price fluctuation such as the one experienced by Neodymium. Information on China's specific endowment of Neodymium is not known and neither is the interest rate. Both Initial S and r in this example are arbitrary values.

At an interest rate of 5% we see that extraction is zero. The price received for the metal is not sufficiently high to warrant extraction. When interest rates are changed to 50% on the other hand, some extraction moves to the first few periods. No extraction occurs in the intermediate periods because the discounting effect is too drastic and prices have not risen high enough to counter that effect. In the final ten periods, when extraction begins again, prices have risen sufficiently to counter the heavy discounting of profit by the interest rate.

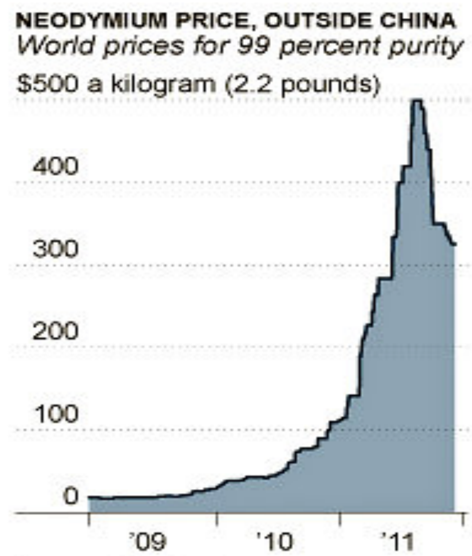

Figure 32: *Prices of Neodymium ($/kg) over the period Jan'09-Jan'12. Source Bradsher (2011).*

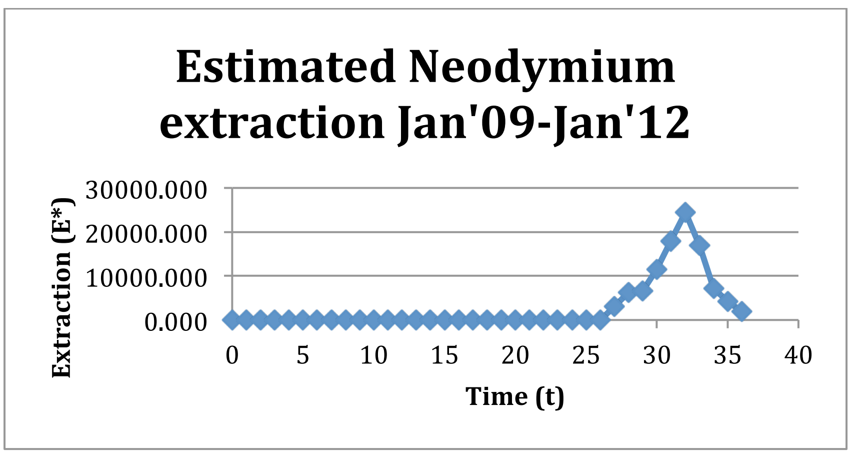

Figure 33: *Estimated Neodymium extraction over period Jan '09 to Jan '12*

Economic Theory and China's REE Extraction

The demonstration in excel, illustrated the theory of optimal extraction using concrete numbers and graphs. It also attempted to model to the extent possible the true case of rare earth elements. Unfortunately, the analysis is limited by the lack of reliable data on rare earth prices and extraction.

The following few pages sum up how economic theory might interpret and predict China's current extraction policy and its behavior in the case of external shocks to price, interest rates or the discovery of previously unknown deposits.

Resource Exhaustion

From the data in sheet *PC Constant Price*, it appears that with 43,000,000 units of the resource and a constant extraction of 130,000 units each year (the last recorded date), the resource will last approximately 330 years. Such a lengthy time period cannot be considered a useful frame for an optimal extraction problem because even at reasonable interest rates (<10% p.a.), the present value of profit from the 300th year would be negligible. Therefore, it appears that the extraction must occur in a shorter period. With this measure of initial stock and extraction, it appears that the problem is not a constrained one as earlier thought.

However, given that rare earths are numerous and have different applications, it is possible that some are scarcer than others (USGS 2010). Light rare earths are more abundant that heavy rare earths but have fewer critical applications. Therefore, it is possible that the heavy rare earths, which are scarcer and have more critical applications, may be close to running out in the next 30-40 years (Tse 2011). In the ideal case, the analysis performed in excel could be run for individual REE compounds or REE metals to account for their very different costs of extraction, prices and demand. Since REEs are sold on private markets however, this information is not available to the public.

Optimal Extraction

It can be argued that China's extraction path is not optimal from the perspective of the economic theory discussed in the previous section. Referring again to the real extraction rates marked in blue on the sheet *PC Constant Price*, we see that extraction was rising steadily in the years before the export restrictions were imposed and mines were forced to shut. This indicates that China was under-producing in those periods and slowly building the capacity to produce at its economic optimal.

However, the economic literature and the discussion on the *Monopoly Price Changes* sheet suggest that monopoly extraction should fall over time because as a scarce resource comes closer to depletion over time, its opportunity cost or scarcity rent begins to rise. From this perspective, and that of real scarcity of REEs in China, it was in China's economic interest to restrict exports of rare earth elements to stem the rise of scarcity rents and preserve the resource for future extraction.

Discoveries of New Stocks

As explored in the sheet *Initial S Comparative Statics*, the discovery of new stocks of REEs in China will increase the amount of extraction that occurs in each period and extend the period over which this extraction takes place. The optimal extraction trajectory will move outward from the origin.

Reserves of REEs could also increase if other countries brought their mines back into production. However, this possibility is not accounted for in the excel workbook because it would entail finding the market equilibrium in a perfectly competitive market, which in turn would require information on market demand and supply, for which data are not publically available. If such data were available, this market could in theory be fully modeled and the effects of increased stocks could be determined.

In the discussion in excel, these prices were assumed to be exogenous, or determined outside the system. The reopening of mines or discovery of new reserves in the United States or Australia would be an exogenous shock to the system generating a new set of lower exogenous prices. Lower prices would lead to less production by individual Chinese producers.

Interest Rate Fluctuations

Changes in the interest rate were discussed in the *r Comparative Statics* excel sheet. The discussion there noted that an increase in the interest rate made extraction more lucrative closer to the present. While this is true, increases in the interest rate could raise the value of a currency relative to others leading to reduced demand for Chinese products, including REEs, which could in turn create an exogenous change in prices. On the excel sheet this would be manifested through a change in the exogenous interest rate and a change in exogenous demand coefficients. The full macroeconomic dynamics of interest rate fluctuations, currency values and resulting REE price fluctuations are not explore in this paper but may be interesting areas of further research.

In addition, a full discussion of interest rates would require a mention of yield curves, which describe the path of interest rates across time. In the context of extraction, the optimal extraction path perceived by a producer would be determined by their estimate of interest rate. However, the true interest rate would only be known from the yield curve if the full period of extraction were known. But this period is known only when the interest rate is known. This creates an endogenous system which would require a theoretically complex discussion to fully understand. That discussion is beyond the scope of this paper.

Externalities, Market Failures and Government Intervention

A market failure is defined as, "an inefficient allocation produced by a market economy" (Tietenberg 2000). The cause of the inefficient allocation is often an externality. This is the cost of an activity that is not included by an actor in their decision-making.

Environmental damage is perhaps the most common kind of externality, where society instead of a private actor bears the cost of environmental degradation (Tietenberg 2000).

In the case of REEs, incomplete enforcement of environmental protection laws has allowed the growth of an illegal mining sector in China which until recently produced much of that country's REE exports. As mentioned in the introductory section of this paper, mining REEs is inherently polluting from the radioactivity of the tailings (USGS 2010). Illegal mines in China, working outside environmental standards for safety, failed to internalize the costs of their operations and therefore overproduced REEs. As a result, the government's export restrictions (quotas and export taxes) could be understood as an attempt to force REE producers to internalize the full costs of the environmental damage that their operations generate.

Assuming that the cost to society of the pollution generated by mining could be measured, then the economically efficient allocation could be determined simply by adding the cost of the pollution to the costs borne by the firms. This would increase the cost parameters (c_0 and c_1) and cause extraction in each period to reduce by an amount dependent on the increase in marginal cost (Tietenberg 2000). In the case of REEs, without specific knowledge of the costs of extraction or the cost of pollution, it is difficult to know by how much the cost parameters should be increased.

Other Economic Theory

A number of other economic approaches could have been used to analyze the REE extraction problem. These are discussed briefly below and could serve as areas of future research.

Monte Carlo Simulation

Monto Carlo simulation is an econometric tool that simulates random numbers as an aid to estimation. In the REE extraction problem, Monte Carlo simulation could be used to simulate random prices while the resulting effect on extraction is observed. Using a program that would perform this simulation thousands of times would chart thousands of slightly different extraction paths across time. The average of these paths would be a good estimation of the true extraction path for REEs.

Estimating a Demand Curve

Another econometric tool that could be used to understand REE markets and extraction paths would be the econometric estimation of a demand and supply curve for REEs towards solving forecasting prices in the case of shocks. This analysis would require extensive research into the derived demand for REEs, the dynamics of those industries, and the existence of possible substitutes. Supply of the element would depend on such factors as the geological availability, state of extraction technology and political factors that might affect supply decisions. Potential hurdles with this approach would be the identification of a REE compound or metal for which to model a market and the availability of sufficient reliable data to estimate equations.

Game Theory

Market equilibrium in the REE markets could also be studied using game theory. Given that there are a few firms in the world that mine and process REEs, an analysis of how their decisions are interrelated would help to forecast the availability of REEs in the case of exogenous changes in price, REE stocks or government regulation. In this case, comparisons could be drawn between the REE oligopoly and OPEC. At present however, China has a monopoly over REEs. The oligopoly analogy will only apply in a few years time, when other firms reach full functionality and can compete with the Chinese monopoly.

While Section 2 has explored some methods of Economic analysis that help to understand REE extraction, it has assumed that the objective of the Chinese government is profit maximization. However, maximizing the monetary value of REEs may not be the sole objective of the Chinese government. International trade relations, domestic politics and adverse environmental effects surely factor into the decision-making, although perhaps in unquantifiable ways.

In Section 3 that follows, this paper develops an analysis of the REE extraction problem from the perspective of Political Science, where strategy may be modified based on non-monetary considerations such as power or national interest.

Section 3: The Politics of Extracting Rare Earth Elements

Section 3: The Politics of Extracting Rare Earth Elements

China and the WTO

Rare earth elements among other scarce resources are increasingly becoming a national security concern, particularly because they have high-technology applications and no close substitutes (Milder & Lauster, 2011). However, in discussions of China's rare earth policy, the complexity of China's domestic politics is largely ignored, a major oversight in a one-party state that nonetheless has a growing civil society presence that it becoming more active.

It is not in China's long-term interests to undermine WTO standards, nor is it in the developed world's interest to alienate China from the WTO and other international institutions. Rather, the international community needs to be more sensitive to China's unique status as a developing country with the problems associated with rapid growth and massive poverty. It also needs to recognize that while China is outwardly a unitary bloc, its burgeoning civil society groups, particularly Environmental NGOs (ENGOs) are becoming more of a domestic force. The following discussion will cast the Chinese government as an actor caught between the international community represented by the WTO and the domestic political field, composed of the needs and challenges arising from development, particularly the challenge of environmental degradation that has been used in defense of restrictions on rare earth exports.

First we discuss the WTO and China's relationship with that institution, framing the dispute over rare earth elements in that context. This section contains the following four components:

(i) China and the WTO benefit each other.

(ii) Development implications of China's accession to the WTO.

(iii) WTO resolution of resource disputes.

(iv) WTO precedents to rare earth dispute.

Subsequently we discuss the growth and effectiveness of ENGOs in China's domestic politics to consider how this might be a factor influencing China's REE export policy.

(i) China and the WTO benefit each other.

In 2001, China joined the WTO after fifteen years of negotiations (Cass, Williams & Barker, 2003). This event marked a milestone in China's process of liberalization, but also heralded new challenges that the country and the WTO would face in accommodating each other.

Accession to the WTO is in China's interest because it speeds up and legitimizes broader economic reforms and through active engagement with the international community retains investors' confidence in that economy (Drysdale & Song, 2000). China's accession came at a time that major economic restructuring was still taking place with high adjustment costs (Drysdale & Song, 2000). Joining the WTO and forced into compliance with its terms of accession, has committed China to its economic reforms and laid a clear timeline in which these reforms were to happen. Additionally, the benefits from trade and efficiency began to accrue to Chinese industry to offset some of the adjustment costs that came from the new economic structures (Drysdale & Song, 2000).

In the years following, China has benefited from export led growth. While the global economic recession slowed growth in the first quarter of 2009, subsequently China's GDP grew at an annual rate of 8.7% (WTO TPR 2010, p vii). China is the world's largest exporter and second largest importer behind Germany and the United States respectively (WTO TPR 2010). This indicates the greater interdependence between China and the rest of the world that has been facilitated through multilateral institutions like the WTO. As the interdependence suggests, members of the WTO have benefited enormously from China's accession to the WTO through access to markets for manufactured goods, raw materials, financial, and legal services (Drysdale & Song, 2000). In addition, the improved transparency and accountability that China faces in the WTO make the country a safer location for investment (Drysdale & Song, 2000).

A major concern arising from China's involvement with the WTO was the mismatch between the WTO's own western and litigious system of dispute resolution and China's legal system. While the WTO's dispute settlement is based on a system of rights, litigation and judicial review, the Chinese system is less transparent and much more decree-based (Cass, Williams & Barker, 2003). Scholars feared that the opacity of China's system and its possible noncompliance with WTO rulings could destabilize the dispute settlement process (Cass, Williams & Barker, 2003). Other scholars stressed that having a different perspective to international law might help to make it more representative of the developing world (Drysdale & Song, 2000).

China's size and political clout are also seen as a threat by some members of the WTO (Drysdale & Song, 2000). It could be argued that this fear of China remains to color the discussion about rare earth elements and China's monopoly of productive capacity in that industry. While China has far to go in terms of liberalizing trade and improving the transparency of its economic processes, this fear is unjustified. Part of China's political opacity arises from its own limitations as a developing nation struggling to enforce standards created by the industrialized world (Magariños, Yongtu & Sercovich, 2003). The country also struggles with development challenges and environmental issues, in a political environment in which information is not freely available and civil society is only now developing. In the next section we discuss what China brings to the WTO as a developing country member.

(ii) Development implications of China's WTO accession.

As a developing country member of the WTO, China faces unique challenges but creates opportunities for the system itself to be changed for the better. China is the most powerful developing country member of the WTO and therefore serves as a spokesperson for developing country interests as it defends its own (Magariños et al., 2003).

During its time of WTO accession, China has been undergoing four types of structural reform that have implications for WTO accession and the case of rare earth elements (Cass, Williams & Barker, 2003). First, the economy has been shifting from a planned economy to a market economy where government planning serves to guide rather than set market outcomes. Second, despite restrictions on private enterprise, this sector has been growing relative to the shrinking public sector. Third, secondary and tertiary industries are growing in favor of agricultural sector of the economy. Fourth, the economy is transforming from a closed to open system (Cass, Williams & Barker, 2003).

Within the broader framework of structural reform, we have a better understanding of the rare earths dispute. Government quotas and export restrictions on the rare earth industry need to be viewed as one part of a system in which the government plays a significant role in directing the growth and relative specialization of the economy's sectors. Similarly the attempt to develop downstream manufacturing in the rare earth industry is part of a broader shift from an economy reliant on its primary sector to one that is stronger in its secondary and tertiary sectors. The country seeks not only to export raw material, but to develop the capacity to process and produce export products higher up in the value-added chain (Tse, 2011). The shift from the production of raw materials to that of high-tech exports is one that most countries have taken on their path of export-led growth.

Another major implication of structural changes to the Chinese economy and to the rare earth's industry is the pace of growth and change. As the government becomes less directly involved in the economy, and the private sector begins to enjoy more freedoms, the pace and direction of growth quickly move out of the government's control (Magariños, Yongtu & Sercovich, 2003). The restrictions placed on rare earth exports could be seen as an attempt by the Chinese government to regain some control over an extractive industry that has huge potential for growth, but also sordid environmental effects if mishandled.

From the perspective of environmentally sustainable development, WTO accession could produce uncertain outcomes. It leads to changes in scale, composition and technology used in industry, which could either be environmentally friendly or not (Magariños, Yongtu & Sercovich, 2003). Increased competition from foreign sources could cause a weakening of environmental protection legislation or enforcement in the interests of international competitiveness (Magariños et al., 2003). The Chinese government claims that before the imposition of the export restrictions, the rare earth extraction in China was getting out of hand, as a result of international demand for the materials that could not be met by licensed mines that operated within environmental regulation (Tse, 2011). This could be understood as a failure of the market to fully internalize environmental costs, and more generally, a result of incomplete structural reform in the Chinese economy.

Developed or industrialized members of the WTO have also been living by a double standard with regard to liberalizing their own international trade regimes. In particular, the protection of agricultural commodities by industrialized countries has been institutionalized under GATT and the WTO and as a result, trade in non-agricultural goods has grown 18 times since 1947 while trade in agriculture has only grown 6 times (Magariños et al., 2003). Non-trade barriers to agricultural trade were to be converted to ad-valorem tariffs. The EU in particular has used loopholes in the law to exclude vast sections of goods from tariffication and provided other forms of support to this sector (Magariños et al., 2003).

The ATC (Agreement on Textiles and Clothing) and MFA (Multi-Fiber Agreement) are two other cases in which the industrialized world has strained norms of international cooperation. These two agreements detailed a time frame and process for liberalizing international textile and clothing trade. While state protection of those industries has gradually been revoked in those countries, it has been such a protracted process that it occurred over a period longer than the period China was allowed to comply with WTO norms (Magariños et al., 2003). China has been the developing country most affected by these barriers to textile trade and it is estimated that fully dismantling the EU's quotas would increase aggregate production in the China's textile and clothing sectors by 8 percent and 59 percent respectively (Magariños et al., 2003).

Intellectual property protections under TRIPS (Trade-Related Aspects of Intellectual Property Rights) are another arena in which the developed world has imposed a morally unjustifiable burden on the developing world. The protection of intellectual property rights of pharmaceuticals in the developed world have for example made medication for AIDS unaffordable for many people in the developing world (Magariños et al., 2003). More generally, TRIPS facilitates a transfer of wealth from the producers and consumers in the developing world to the developed world through rents on intellectual property, a transfer of wealth that is inconsistent with targets for human and economic development (Magariños et al., 2003). The E.U. is attempting to have pharmaceutical companies sell drugs at reduced prices to the developing world, but this represents "more of a patchwork than a serious multilateral reassessment" of the negative effects on TRIPs on international development (Magariños et al., 2003, p. 151).

In the WTO, the industrialized world has been guilty of setting "international legal standards" with little consideration of the difficulty of administering these or their effects in the developing world (Magariños et al., 2003, p.153). Part of the reason that these international agreements are not representative of the development perspective is because after the Uruguay Round of WTO negotiations, the developing world has been either excluded from meetings or provided incomplete information, therefore "negotiating blindfolded" (Magariños et al., 2003, p.157). As a result, this bloc of countries has become increasingly disillusioned with the benefits that the world trade system can accord it.

China can use its clout in the WTO to demand processes that allow the full participation of the developing world in the framing and execution of WTO decisions. This is likely to strengthen rather than weaken the global trading regime (Magariños et al., 2003).

iii) WTO settlement of resource disputes.

An understanding of international law regarding trade restrictions and particularly the manner in which the WTO interprets these laws has bearing on the general trade in scarce resources and the particular case of rare earth elements. In March 2012, the U.S., E.U., and Japan filed a dispute in the WTO against China's restrictions on rare earth exports (Bradsher, 2012). This section will detail the reasons for imposing restrictions, what restrictions may be lawful and the defense that China may present in its rare earths case.

Multilateral trade law prohibits quantitative restrictions, like quotas, on trade. However export taxes are banned in only a few cases in which they fail to apply equally to all export markets. In addition, multilateral trade law makes many exceptions to the law for reasons of national security and environmental protection among other things (Milder & Lauster, 2011).

Export restrictions may take the form of taxes, duties, quotas, export bans, reductions of VAT rebates or stringent export licensing systems that raise the administrative cost of international trade (Milder & Lauster, 2011). Of these, export taxes are the kind used most often, levied ad valorem (as a percentage of value) or specific (per unit or weight of product) (Milder & Lauster, 2011). Export quotas restrict the quantity that can be exported within a given period of time.

WTO law does not prohibit export taxes, but such taxes must be "non-discriminatory and transparent" as described by Articles I and X of GATT(1994) (Milder & Lauster, 2011, p.262). Export taxes provide some flexibility because if they are reduced, they can be increased again in the future without violating WTO law. This is not the case for quantitative restrictions on exports (Milder & Lauster, 2011)

A few main motivations for restricting exports are to help infant industries get a foothold, promote income redistribution, augment government revenues, protect the natural environment and conserve resources (Milder & Lauster, 2011). The U.S. International

Trade Commission finds that export taxes are typically used by low and middle-income countries to protect domestic industry and generate revenue for the government. Quantitative restrictions, like quotas, are usually imposed by high-income countries for national security, or environmental protection and low-income countries usually impose quantitative restrictions to protect the environment or public health (Milder & Lauster, 2011).

GATT allows for exceptions to the law under certain conditions that are described below:
1) GATT XI (governing critical shortages) allows for quantitative restriction if they are
i) "temporarily applied to relieve critical shortages of foodstuffs or other products"
ii) "necessary for the marketing of commodities" (Milder & Lauster, 2011, p. 264).
2) GATT XX provides exemptions from WTO obligations if they are:
b) "…necessary to protect human, animal or plant life and health…"
g) necessary for the "conservation of exhaustible natural resources" (Karpinar, 2011, p. 401).
i) necessary for price stabilization.
The exception does not apply to protect domestic industry.
3) GATT XXI makes allowances for national security objectives, restrictions in war or other times of international emergency. It is unclear if this clause applies only to political emergencies or social and economic emergencies too. (Milder & Lauster, 2011)

The UN Treaty on Non-Proliferation of nuclear weapons and UN treaty on the development and trade of Chemical Weapons are examples of restrictions on trade for national security reasons. CITES (UN Convention on International Trade in Endangered Species of Wild Fauna and Flora) is an example of trade restrictions for the purposes of protecting the natural environment (Milder & Lauster, 2011).

In the event of instituting export restrictions, Article 12 of Agreement on Agriculture requires that the country restricting exports inform the Committee on Agriculture in writing, and consult with countries that would be significantly affected by the limit on imports (Karpinar, 2011).

Though export restrictions are sometimes legal under the WTO, most economists would agree that the only valid justification of export restrictions is food shortages. Otherwise, export restrictions risk international trade distortions and are, "a second-best policy tool to address domestic market failures" (Milder & Lauster, 2011, p. 253). Export restrictions redistribute income from raw material exporters to downstream industry and in the worst case encourage the development of substitutes to the raw material and retaliation from trade partners (Milder & Lauster, 2011).

All members of the WTO are not bound by the same laws and obligations. Members of the WTO can agree to more stringent, legally binding commitments in their individual accession treaties. Bulgaria, Ukraine, and Vietnam have done this. However, China's treaty of accession is the most stringent of late member states (Karpinar, 2011). Besides applying export restrictions to 84 items listed in the Annex of the Accession Agreement, China has committed to applying no other export restrictions (Karpinar, 2011). Perhaps because of the more stringent than usual accession treaty, China is often accused of being in violation of its WTO obligations.

China is placing a variety of restrictions on its exports of rare earth minerals. In late 2011, the export quotas for 2012 were modified to differentiate between heavy and light rare earths and while overall quotas were relatively unchanged the split reduces the quota allotment of rarer heavy REEs (Yap, 2011).

Year	Export Quotas	Percent Change year on year	Estimated non-Chinese demand
2004	65609		57000
2005	65609	0.00	46000
2006	61821	-5.77	50000
2007	59643	-3.52	50000
2008	56939	-4.53	50000
2009	50145	-11.93	35000
2010	30258	-39.66	-
2011	30184	-0.24	-

Figure 1: *REE Export quotas from 2004-2011. Data from Kim & Korinek (2010). Export quotas for 2010 and 2011 from Yap (2011).*

We see that after the sharp reduction in quota allotments for 2010, export quotas have not been changed much. In addition to export quotas, export tariffs are levied ad valorem, and mining firms are required to be licensed for export (Kim & Korinek, 2010).

Material	Tax Rate
Neodymium metal	15%
Other rare earth metal	25%
Europium, Terbium, Dysprosium, Yttrium (Oxides, Carbonates, Chlorides)	25%
All other rare earth oxides	15%
Ferro rare earth alloys	20%

Figure 2: *Export taxes on REE exports effective from January 2008. Derived from Korinek & Kim (2010).*

As a result of export quotas and taxes, non-Chinese processors of REEs pay approximately 31% more for REE raw material in addition to the costs they incur for transport and storage (Kim & Korinek, 2010).

In the ten years since it acceded to the WTO, China has been accused in a number of cases. In 2007, China imposed export restrictions by eliminating value-added reseller (VAR) rebates on products regarded as highly polluting or energy or raw material intensive (Milder & Lauster, 2011). In 2009, the U.S.A. and Mexico filed a case against China's restrictions of the exports of raw minerals (Karpinar, 2011). This case, *China Raw Materials* does not concern restrictions of rare earth exports, but the outcomes of the case have implications for China's REE case. In January 2012, the W.T.O. ruled China's export restrictions unlawful (Bradsher, 2012). In March 2012, a formal case was filed against China's restriction of the exports of REEs (Bradsher, 2012).

However China is not the only country to impose export taxes illegally. Russia maintains a 5 percent export tariff on copper scrap. India imposes a 15 percent tax on iron ore and the Ukraine restricts 24% of aluminum scrap trade. Venezuela has a complete ban on the export of copper, lead and cobalt scrap (Milder & Lauster, 2011)

All these cases go through the WTO dispute settlement procedure. The process is generally considered just and efficient in resolving trade disputes for the following reasons.

- Procedural rules are set – The dispute settlement process is preset and adjudicative rather than ad-hoc. As a result, it is seen as a fair process for all countries regardless of size.
- A time frame exists for each stage of the dispute settlement process such that the process cannot be dragged on for an indefinite period of time. The parties to each case have time-bound obligations.
- A responding country cannot refuse to be judged. Member countries that have been brought to the dispute settlement body cannot refuse to participate in the process.
- Cases are reviewed by an "independent dispute panel, usually chosen in consultation with the countries in disputes" (Milder & Lauster, 2011, p. 267).
- Appeals are possible, but they must have a legal basis (such as interpretation)
- Decisions are legally binding once they are adopted. Failure to comply can result in sanctions from plaintiffs (Milder & Lauster, 2011).

Nonetheless, the number of exceptions available under GATT, some of which were described earlier, leave much room for interpretation and differences of interpretation. Using a weak dispute settlement process that is opaque or unclear could make the dispute settlement process appear politically rather than legally motivated and thus alienate developing world from the WTO (Milder & Lauster, 2011).

In addition, even while decisions and the actions of parties to a dispute are time bound, countries that cheat on their obligations can make short-term political and economic gains. Whether China's restrictions on rare earth exports are such a case remains to be seen. In the short run, while other countries have few alternate sources of rare earth minerals, it appears that the WTO is powerless to prevent China from accruing resource rents and using its monopoly power. The following section will detail some WTO disputes which may serve an important precedent in the WTO rare earth case.

(iv) WTO Precedent to Rare Earth Dispute

EU against Argentina "Export Restrictions on Hides and Bovine Leather", 1998

In this case the E.U. complained against Argentina on two counts: (i) Representatives of Argentine leather tanning industry needed to be present during customs procedures for exports for the disclosure of information about slaughterhouses for hides and leather (this allegedly violated GATT XI)
ii) Advance tax payments imposed a higher burden on imports (Milder & Lauster, 2011).

The WTO dispute settlement panel did not find that the administrative procedure constituted a trade restriction under GATT XI, however it did find that it was not implemented in a "reasonable and impartial manner" (Milder & Lauster, 2011). It also failed to guarantee the confidentiality of information made available through the process. The panel asked Argentina to modify its policies by Feb 2002, which it did (Milder & Lauster, 2011).

This dispute illustrates the "reasonable and fair" clause of GATT XI and the importance of transparency and uniformity in the application of export restrictions, particularly administrative or taxation. In the rare earths dispute, China has been accused of being unreasonable and non-uniform in the restrictions it has placed. This may be found to violate its treaty agreements (Milder & Lauster, 2011).

Canada against the U.S. "Measures Treating Export Restraints as Subsidies", 2000

The case explored whether export restraints could be considered a form of subsidy. This applies to the China REE case because export restrictions could subsidize downstream industry.

Here Canada restricted the export of lumber to the U.S., which, under U.S. law qualified as an export subsidy, justifying countervailing measures by the U.S. The U.S. alleged that the export restrictions were a form of subsidy, while Canada justified them as "financial contribution" under U.S. law and practice (Milder & Lauster, 2011).

At the core of the problem was a disagreement between Canada and the U.S. on their definition of an export subsidy. The WTO panel ruled in the U.S.'s favor, but the case was a sign that the WTO had unclear definitions of subsidies, and more generally, a legal system that was opaque. This challenges the fairness of that institution's legal rulings.

U.S., E.U and Mexico against China: "Export Restrictions on Metals" (China Raw Materials Case), 2009

In 2009, the three entities above filed a case against China in the WTO. They claimed that China's quotas and export taxes violated WTO agreements. A number of countries joined as 3^{rd} parties to the case.

The following three complaints arose:
(i) Quota restrictions on the export of bauxite, coke, fluorspar, silicon carbide, and zinc are unlawful under article XI of GATT 1994.
(ii) Temporary and "special" duties of different magnitudes on the materials above are a violation of the accession protocol – which restricts such duties to 84 materials (none of the above).
(iii) Administration of the export restrictions is not "uniform, impartial and reasonable". Also, China is not transparent about its restrictions and has failed to make public some of its requirements, restrictions, and export prohibitions (Milder & Lauster, 2011).

China justified its export restrictions on the basis of the following:
(i) Conservation of natural resources
(ii) Environmental protection and energy saving

(iii) Ensuring stable domestic supply

(iv) Managing trade to reduce the current account surplus (WTO 2012)

WTO Trade Policy Review in 2010 alleged that China may be giving its downstream manufacturers an unfair advantage. This does not constitute a legal ruling, but only an indication of the legal frailty of China's defense (WTO, 2012).

In January 2012, the WTO dispute settlement body ruled that China's restrictions on the exports of raw materials were inconsistent with its accession obligations (Bradsher, 2012). Specifically, China's accession agreement overruled GATT XX, which justifies export restrictions on the basis of environmental protection (WTO 2012). Also, since China has not made attempts to reduce demand and refining of rare earth elements domestically, its application of the restrictions were also found to be non-uniform, in a violation of its accession agreement and the cases under which GATT XX may be applied (WTO, 2012). China was asked to bring its restrictions in line with WTO guidelines and its accession agreement.

This case serves as precedent for the case filed in March 2012 by the U.S., E.U. and Japan against China's rare earth export restrictions (Bradsher, 2012). That case is similar in that China uses GATT XX to justify the export restrictions. Since GATT XX was overruled by China's accession agreement, it is unlikely that it will be upheld in the REE trade dispute. However, given China's requirement that rare earth exports be environmentally certified, the WTO body may find that the restrictions are uniformly applied and affect the domestic as well as international markets. In addition, the Chinese government has begun nationalizing larger rare earth mining companies (Bradsher, 2012). If this goes through, the REE industry in China could become a state owned oligopoly and implement export restrictions without explicit government policy. The WTO is only capable of ruling on government policy and has little authority over oligopolies (Bradsher, 2012).

Altogether, an analysis of the WTO ruling in the China raw materials case is inconclusive in its precedent for the REE case. In any case, by the time the WTO verdict is out, China's REE industry may be a government owned oligopoly and manufacturing firms would likely have shifted their operations to China from Germany or Japan, serving the Chinese governments interests to develop downstream industries for REEs (Bradsher, 2012).

REEs & China's Environmental Movement

Part of the reason that developing downstream industry is so attractive is because it is not as harmful to the environment as mining. As China engages internationally and develops, the condition of its environment is deteriorating (Drysdale & Song, 2000). The World Bank estimates that environmental damage in China amounts to 8.0 % of GNP. Abatement costs would be about 1.6% of GNP (Magariños et al., 2003).

Yet Chinese environmental policy has slowly taken shape. Since the 1980's the State Environmental Protection Agency has required Environmental Impact Assessments (EIAs), reviewed pollution control equipment and monitored operations of new plants, in addition to having concentration-based guidelines on pollution (Magariños et al., 2003). Unfortunately, as laws to protect the environment have expanded, they have continued to remain unenforced (Magariños et al., 2003). Part of the reason that Chinese environmental bureaus have been unable to enforce environmental regulations is because local officials are evaluated on economic growth parameters and therefore direct their attention to the short-term rather than long-term sustainability practices (Yang, 2005).

In response, the Chinese government has encouraged the growth of a non-government sector to tackle social issues as a "third force" (Yang, 2005, p. 54). The regulation, registration and management of these organizations is clearly defined by law but there is a clear understanding in the government that it cannot monitor or enforce as well as civil society can through citizen groups.

In the past twenty-five years, non-state economic activity has been the primary stimulus for greater openness and reform in international engagements (Morton 2005). However, on the issue of the pollution from extraction and trade of rare earth elements Chinese ENGOs appear to be working in opposition to the international community. Chinese ENGOs are working to develop a greener, more sustainable economy while the rest of the world appears to want China to continue to mine REEs at the low costs that come from poor environmental regulation. This has implications for the future of citizen participation and perception of international engagement in the domestic community.

NGO's in China are growing in influence because they engage in politics, media, the internet and international NGO's – this gives them the ability to mobilize more quickly with fewer resources (Yang, 2005). Web-based NGOs are able to avoid resource constrains and political constraints a little easier than others (Yang, 2005). They focus on publicity and encouraging participation rather than opposing the government. In this sense the system is very much like the European corporatist interest group representation, where civil society groups work in cooperation with the government (Yang, 2005). The systems are also similar in that only one organization may speak for a particular issue in each administrative area. Often GONGOs (Government owned NGOs fill these single representative spots (Yang, 2005).

Still at an early stage of their development, NGO's choose "non-confrontational methods" to achieve their objectives (Yang, 2005, p. 53). The agenda of these organizations, even as seen by those within them is purely environmental and completely apolitical. Yet, some scholars believe that China's NGOs function as a "barometer of political change within Chinese society" (Morton, 2005, p. 519). ENGOs in China serve as training grounds for democracy. Through these organizations Chinese civil society learns the political skills, civic participation and learns to "test the boundaries of political control" (Yang, 2005, p. 65). NGO's play an important role in Chinese politics to educate the community on the issues and the rule of law, stimulate public participation, combat corruption by "increasing transparency and accountability" (Morton, 2005, p. 526).

However information disclosure programs are an area in which China does particularly poorly, placing limitations on the effectiveness of China's NGOs.

In the developing world information is available to civil society which can then exert pressure on firms through NGOs. In China, this information is not made available to the public and NGOs have restricted freedoms to monitor and enforce laws. The result is little public pressure to change things (Magariños et al., 2003).

Yet China's NGOs have some factors that bolster their activist political power. First, NGO's enjoy a "homologous" relationship with the media since both operate within government control but have a significant degree of autonomy. Media organizations and environmental groups share similar moral and political interests that fit the communist party's official policy of sustainable development, making their demands and dissemination of information politically safe (Yang, 2005). Its alliance with the media gives China's NGOs leverage in society through the ability to widely broadcast messages and mobilize society. Second, small and large NGOs alike receive international funding and form a part of transnational NGO networks. These international networks legitimize their activities and therefore give them more influence and the resources to act in the local arena (Yang, 2005).

One example of the effectiveness of Chinese NGOs is the case of the damming of the Nujiang river, the last free-flowing river in South Asia (Morton, 2005). The damming of this river would have resulted in the loss of biodiversity and 22 cultural minorities who lived along the river. Local officials ignored the obvious environmental and social issues that would arise from the dam project being completed in the interests of economic development. Working with international NGOs like Greenpeace and the media, local NGOs were able to publicize the harm that this project would generate sufficiently that Premier Wen Jiabao ordered that the project be halted until further environmental assessment tests could be completed (Morton, 2005). While local officials are biased in favor of economic growth over environmental sustainability, high ranked officials and

Chinese official policy recognize that environmental sustainability is a priority (Diamond & Liu, 2005).

The case of REE extraction can be understood as one in which the Chinese government is caught between Chinese civil society which is becoming more powerful and the international community in the WTO. ENGOs would like mining to be regulated or halted in order to make China's growth and development environmentally sustainable while the WTO must uphold international trade law and hold China to its accession protocol. The Chinese government is trying to win on both of these fronts. On the domestic front it speaks of sustainable development and begins to close down illegal mines and require the environmental licensing of REE exports. Meanwhile on the WTO front it uses GATT provisions for environmental protection to side-step its accession commitments.

It is a fortunate coincidence that China's own national interest lies with the domestic front. Restricting rare earth exports helps to make REE mining less polluting, allows miners to earn scarcity rents, and puts pressure on international manufacturers to shift downstream production to China. This shift of downstream production gives China more control over not just the raw material but also the critical finished products, engines and permanent magnets in which REEs are used. The shifting of plants also allows the possibility of technology transfer and the rise of Chinese manufacturing of these high-tech products. In the longer-run this is in China's interests and follows the export-led development strategy of South East Asia in which primarily agrarian economies shifted to the export of manufactured goods and technology.

As discussed earlier, the WTO ruling on China's REE export restrictions may come too late for the international community since firms are already moving their manufacturing to China and researching REE substitutes and recyclability. While in the short-run, the issue of REEs has seemed critical, in the longer-run it appears that prices are falling, there is no real shortage of REEs, and China may be developing a greener domestic mining and manufacturing sector. A clean, green, and developed China is in the interest of all nations and may serve to strengthen the WTO and international cooperation, both in the challenges that it poses along the way and the hope that once China is fully transitioned it will be more capable of abiding by the WTO's standards.

Conclusion

The economic theory of optimal extraction and the excel demonstration were useful as a guide to the extraction of rare earth elements and other depletable resources. However, the lack of reliable data on the price and traded quantities of specific elements mean that the models are only partially calibrated and therefore not useful charts of 'real' extraction paths. Were data to become available in the future, this method could be used to form some preliminary forecasts of how China's producers may behave. The methodology could still be used as an aid to teaching theories of optimal extraction and perhaps applied to other depletable resources for which data are available.

The analysis of China's relationship with the WTO and its environmental movement suggest that the restrictions on REE exports must be viewed within the context of the country's leadership caught between the demands of international and domestic forces that it seeks to balance as it transition from a developing nation with a state-directed economy to an industrialized, market economy. In this case, it is in the interest of all nations to hold China to its WTO obligations while begin cautious not to alienate it from that forum or others in which international trade and cooperation flourish.

Future research on REEs might apply Monte Carlo Simulation to the analysis of extraction paths or model demand curves for specific elements to provide more accurate estimates of prices. Game theory could be used to study the behavior of the REE market modeled as an oligopoly. In the field of politics, it would be interesting to consider REEs and other natural resources as they are used as leverage in international relations to redistribute power. On the domestic front, the effect of natural resource heavy development on China's internal politics, particularly its rural-urban divide, would be interesting to explore.

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